Pricing and Hedging a Hybrid Pension Plan

by

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Abstract

A defined contribution plan with a defined benefit minimum guarantee is valued as a put option or exchange option. We compare the resulting price with the actual additional contribution rate used by one public sector plan, and consider extensions of the option theory to allow more accurately for the pension plan characteristics.

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Chapter 1
Background

A hybrid pension plan offers the advantages of a money purchase pension plan, but with a guaranteed minimum pension benefit to protect members against adverse investment experience.

The Money Purchase benefit accumulates contributions paid by members (and also by their employers) in individual member accounts. When a pension plan member reaches retirement age, the money in the account may be used to buy a pension for the member, and his or her spouse.

The amount of pension that is purchased with the Money Purchase component account will depend on investment performance and other economic factors. The guaranteed minimum benefit ensures that, even if the economic factors are not favorable, the pension will not fall below a fixed amount which depends mainly on the members' length of service in the pension plan, and their salary in the years before retirement.

To further understand the benefit, let us look at a real case: York University's Pension Plan (YUPP, 2004). Full employees who are at least 30 years old are required to join York University's pension plan. It requires their members to pay 4.5% of basic salary below the YMPE (Yearly Maximum Pensionable Earnings, which is determined by Canada Pension Plan; the value in 2004 is $40,500), plus 6% of basic salary above the YMPE to their Money

Purchase Account. The University will contribute the same amount to the money purchase component account for each member. For a faculty member earning \$50,000 per year, the annual contribution would be:

4.5% on the first \$40,500	= \$ 1822.50
+6% of \$9,500	= \$ 570.00 (9,500 is the salary above 40,500)
Member's annual contribution	= \$2392.50
University's contribution	= \$2392.50
Total	= \$4785.00

All the money purchase contributions are paid into a trust fund. The fund is invested in a diversified portfolio of equities and bonds. At retirement, the member's money purchase component account will be applied to fund an annuity; this will depend on actuarial factors, such as interest rates and mortality assumptions at retirement. The resulting pension will be compared with the guaranteed minimum benefit. If the pension from the money purchase component account is less than the guaranteed minimum pension, then a supplementary amount will be paid to bring the pension up to the guaranteed minimum benefit.

The guaranteed minimum benefit formula depends on the Canada Pension Plan threshold, the YMPE, which is currently \$40,500 (in 2004). It also depends on the final average earning (FAE), which is the average of the highest five years of earnings before retirement. The benefit formula is:

$$(1.4\% \times \max(FAE, YMPE) + 1.9\% \times \max(FAE - YMPE, 0)) \times \text{years in service}$$

For example, suppose a plan member retires with 35 years' service, with final average earnings of $100,000. The guaranteed minimum benefit per year of service would be 1.4% of $40,500 + 1.9% of ($100,000-$40,500) = $1,697.50. So the guaranteed minimum annual pension would be $1,697.50×35 = $59,412.50. If the pension from the money purchase component account is less than this, a supplementary pension is paid to give a total pension equal to the guaranteed minimum. If the pension from the money purchase component account is greater than the guaranteed minimum benefit pension, then no supplementary pension is paid.

Chapter 2
Basic Model

We can easily show that the hybrid pension plan is similar to a put option. Because we can view the fund in the Money Purchase Account as the underlying asset, and the guaranteed minimum benefit as the strike price, then the payoff of the hybrid pension plan guarantee is the same as a put option:

$$\max(F_{xr}, G_{xr}\ddot{a}_{xr}^{(12)}) = F_{xr} + \max(0, G_{xr}\ddot{a}_{xr}^{(12)} - F_{xr}) \qquad (1)$$

where F_{xr} is the accumulated fund in Money Purchase Account at retirement age,

G_{xr} is the guaranteed minimum benefit per year, and

$\ddot{a}_{xr}^{(12)}$ is the actuarial factor at retirement age (paid monthly in advance)

If the hybrid pension plan guarantee only applies at the normal retirement age, then it simplifies to a European put option; if it can terminate early (due to death, withdrawal, or early retirement), it becomes an American (or Bermudan) option. However, we should also notice that there is a significant difference between American option and hybrid pension plan. Under an American option, we expect the option to be exercised to maximize its value. However, this will not be the major factor in an employee’s decision to take early retirement.

From, for example, Aitken (1996) or Anderson (1990), the general formula for calculating the guaranteed (DB) benefit is:

$$G_{xr}\ddot{a}_{xr}^{(12)} = \alpha \times FAE \times n \times \ddot{a}_{xr}^{(12)} \qquad (2)$$

where FAE is the employee's final average earnings,

α is the accrual rate per year for FAE,

n is the number of years in service, and

xr is the employee's retirement age

From equation (2), we can see that since α is fixed when the plan is established, then the strike price $G_{xr}\ddot{a}_{xr}^{(12)}$ depends on the employee's credited service, final average salary, actuarial factor at retirement age, and also the inflation rate, since the YMPE is adjusted by the inflation index annually.

On the other hand, the fund in Money Purchase Account (the underlying asset value F_{xr}) is also stochastic because the return on the invested capital is random.

$$F_{xr} = c \times S_{xe} \times {}^{s}\ddot{s}_{xe:\overline{xr-xe}|}^{(\tau)^*} \qquad (3)$$

where

$${}^{s}\ddot{s}_{xe:\overline{xr-xe}|}^{(\tau)^*} = \prod_{i=xe}^{xr-1}(1+r_i) + \frac{S_{xe+1}}{S_{xe}}\frac{l_{xe+1}^{(\tau)}}{l_{xe}^{(\tau)}}\prod_{i=xe+1}^{xr-1}(1+r_i) + \frac{S_{xe+2}}{S_{xe}}\frac{l_{xe+2}^{(\tau)}}{l_{xe}^{(\tau)}}\prod_{i=xe+2}^{xr-1}(1+r_i) + \ldots\ldots\ldots$$

$$+(1+r_{xr-1})\frac{S_{xr-1}}{S_{xe}}\frac{l_{xr-1}^{(\tau)}}{l_{xe}^{(\tau)}} \qquad (4)$$

and c is the normal contribution rate,

xe is the employee's entry age,

r_i is the investment return rate in year of age i to $i+1$,

S_i is the salary in year of age i to $i+1$, and

s_i is the salary growth rate in year of age i to $i+1$

Note that in later computations, we ignore mortality from payment date. With no mortality after the contribution payment date, F_{xr} will be a little bit higher than if we allowed for mortality, but this has little effect on the guarantee payoff , $\max(G_{xr}\ddot{a}_{xr}^{(12)}-F_{xr},0)$, and is a common simplifying assumption in pension plan valuation. In many hybrid plans the guarantee does not apply on early exit. In others, only early retirement benefits are eligible for the guarantee.

From equations (3) and (4), we can see that the underlying asset value F_{xr} is random, depending on two random processes: the annual investment return rate and the annual salary increase rate.

Let's recall York University's pension plan described in chapter 1. We have following information: xr =65, FAE is the average of the **highest** five years of earnings before retirement, $\alpha_1 = 1.4\%$ for final salary below YMPE, $\alpha_2 = 1.9\%$ for final salary above YMPE, $c_1 = 9\%$ for salary below YMPE, $c_2 = 12\%$ for salary above YMPE. To simplify this model, we assume $\ddot{a}_{65}^{(12)} = 10$, $\alpha = 1.7\%$ (accrual rate for total final salary), $c = 10\%$ (contribution rate per year), FAE is the average of the five years of earnings before retirement, and there are no pre-retirement exits.

Under the traditional actuarial approach, it is assumed that both the investment return rate and the salary increase rate are constant. Here we give the numerical results for two example members: the first one enters the pension plan at age 30, with entry salary \$50,000; the other one enters the plan at age 35, with entry salary \$60,000.

Table 2.1 Entry Age = 30, Entry Salary= $50,000

Salary increasing rate	Investment return rate	$G\ddot{a}_{65}^{(12)}$	F_{65}	$\max\left(G\ddot{a}_{65}^{(12)} - F_{65}, 0\right)$	Min (c)
3%	6%	$766,757	$860,760	$0	8.91%
	8%	$766,757	$1,292,920	$0	
	10%	$766,757	$1,986,959	$0	
4%	6%	$1,045,254	$991,099	$54,154	10.55%
	8%	$1,045,254	$1,463,299	$0	
	10%	$1,045,254	$2,214,332	$0	
5%	6%	$1,420,947	$1,150,138	$270,809	12.35%
	8%	$1,420,947	$1,668,479	$0	
	10%	$1,420,947	$2,484,506	$0	

Table 2.2 Entry Age = 35, Entry Salary= $60,000

Salary increasing rate	Investment return rate	$G\ddot{a}_{65}^{(12)}$	F_{65}	$\max\left(G\ddot{a}_{65}^{(12)} - F_{65}, 0\right)$	Min (c)
3%	6%	$680,308	$703,040	$0	9.68%
	8%	$680,308	$989,547	$0	
	10%	$680,308	$1,416,373	$0	
4%	6%	$883,669	$795,030	$88,639	11.11%
	8%	$883,669	$1,104,720	$0	
	10%	$883,669	$1,562,661	$0	
5%	6%	$1,145,159	$904,105	$241,054	12.67%
	8%	$1,145,159	$1,239,994	$0	
	10%	$1,145,159	$1,732,825	$0	

Note that $\max(G\ddot{a}_{65}^{(12)} - F_{65}, 0) > 0$ means York University needs to contribute supplementary fund to meet the guaranteed minimum benefit. It is also the payoff of the European put option with maturity at the normal retirement age.

In addition, we are interested in the minimum contribution rate that will ensure the pension plan has sufficient funds to pay for the guaranteed minimum benefit. If we assume the investment return rate of the Money Purchase Account is just the risk free rate (6%), the minimum contribution rate is showed in the last column.

From the tables above, we can get a rough idea of the cost of this method. When the salary growth rate is high and the investment return is not favorable, the guaranteed minimum benefit will exceed the funds in Money Purchase Account.

To fund the guarantee, York University contributes annually additional 3% of the members' required contributions and any additional contributions to fund the minimum guaranteed benefit as certified by the actuary (YUPP, 2004). In other words, York University values the put option cost as 3%×5% = 0.15% of employee's salary. We are interested in two questions: Is the additional contribution rate sufficient? And what is the optimal hedging strategy for the hybrid pension plan?

Sherris (1995) applied a contingent claims valuation approach to option features in retirement fund benefits. His result shows that deterministic actuarial valuation understates the cost of benefit by as much as 25 to 35 percent.

In this paper we use a similar approach, but with parameters and benefits updated to approximate the York University situation. We assume stochastic models for the salary growth rate and investment return rate, and apply several option pricing models to value the put option price, including a deterministic actuarial method, exchange option price model, stochastic actuarial method, equilibrium pricing model and "risk-neutral" Monte Carlo simulation.

Chapter 3

Pricing by Deterministic Actuarial Method

In the traditional actuarial method, both salary growth rate and investment return rate are in P-measure, i.e. the real-world measure, while Q-measure means the risk-neutral measure. All parameters in the traditional actuarial method are deterministic. So we assume both σ_1 (the volatility of the annual salary growth rate) and σ_2 (the volatility of the annual investment return rate) to be 0. Clearly this is an unrealistic assumption; both r_t and s_t are random and they are positively correlated. Based on *Report on Canadian Economic Statistics 1924-2003* (2004) and York University's financial statements from 1994 to 2003 (YUPP, 2004) we find $\mu_1 = 2.7\%$, $\mu_2 = 7.5\%$, where μ_1, μ_2 are the mean values of the annual salary growth rate and the annual investment return respectively.

Following the Entry Age Normal method (Aitken 1996), the option prices as a percentage of salary for a new entrant at age 30, 35, 40, 45, 50 are given in Table 3.1.

Table 3.1 Option Price (as a Percentage of Salary) according to Entry Age under Deterministic Actuarial Method

	Scenario	s	r	30	35	40	45	50
1	Rounded	3.0%	7.0%	0.00%	0.00%	0.00%	0.86%	1.33%
2		3.0%	6.0%	0.00%	0.00%	0.49%	1.91%	2.28%
3		3.0%	8.0%	0.00%	0.00%	0.00%	0.00%	0.44%
4		4.0%	8.0%	0.00%	0.00%	0.00%	0.60%	1.04%
5	Prudent	4.0%	6.0%	0.55%	1.11%	1.71%	2.70%	2.95%
6	Real	2.73%	7.78%	0.00%	0.00%	0.00%	0.0%	0.46%

Scenario 1 uses rounded values with s =3.0% and r =7.0%. For comparison, we construct five further scenarios. In scenario 2, r decreases by 1%; in scenario 3, it increases by 1%. In scenario 4 we keep the gap between s and r to be 4% but increase them both by 1%. The results show two trends: firstly, the option price will increase as new entrants join the plan at older ages; secondly, the cost of the put option will increase dramatically as the gap between s and r decreases.

Scenario 5 represents a ‘prudent’ valuation basis. The result is a substantial increase in cost.

Scenario 6 represents the best estimation value, based on historical returns from York University’s pension plan documentation. We can see that the option price is low and the current contribution rate (0.15%) seems sufficient under this method. But in fact, this method understates the option price.

The main reason why the deterministic actuarial method understates the option price is that it ignores the volatilities of s and r. In addition, from these scenarios we can see the option price is very sensitive to the deterministic assumptions for s and r. For an actuary, choosing the scenario is somewhat arbitrary. So it is difficult to determine the additional contribution rate required in this method.

Chapter 4
Pricing by Exchange Option

An option to exchange one asset for another, first developed by Margrabe (1978), is also called an Exchange Option. It gives the holder the right that at expiration, he can give up an asset worth S_2 and receive in return an asset worth S_1. The payoff is $\max(S_1 - S_2, 0)$, which is the same as the hybrid pension plan's payoff function.

Margrabe assumed both S_1 and S_2 follow geometric Brown motion with volatilities β_1, β_2, and that the instantaneous correlation between them is ρ.

$$\frac{dS_1}{S_1} = \alpha_1 dt + \beta_1 dz_{1t} \qquad (5)$$

$$\frac{dS_2}{S_2} = \alpha_2 dt + \beta_2 dz_{2t} \qquad (6)$$

In Black- Scholes framework, he derived the option price at time zero as:

$$p(S_1, S_2) = S_1 N(d_1) - S_2 N(d_2) \qquad (7)$$

where

$$d_1 = \frac{\ln \frac{S_1}{S_2} + \frac{\beta^2}{2} T}{\sigma \sqrt{T}}, \qquad d_2 = d_1 - \beta \sqrt{T}$$

$$\beta = \sqrt{\beta_1^2 + \beta_2^2 - 2\rho \beta_1 \beta_2} \qquad (8)$$

We should notice that these formulas are independent of the risk-free rate r. This is because as r increases, the growth rate of both asset prices in a risk-neutral world increases, but this is offset by an increase in the discount rate.

In York University's pension plan, the fund in Money Purchase Account and the minimum guaranteed benefit follow the diffusion processes:

$$\frac{dG_t\ddot{a}_{65}^{(12)}}{G_t\ddot{a}_{65}^{(12)}} = \frac{dG_t}{G_t} = \frac{dS_t}{S_t} = \frac{ds_t}{s_t} = \mu_1 dt + \sigma_1 dz_{1t} \quad (9)$$

$$\frac{dF_t}{F_t} = (\mu_2 + c\frac{S_t}{F_t})dt + \sigma_2 dz_{2t} \qquad (10)$$

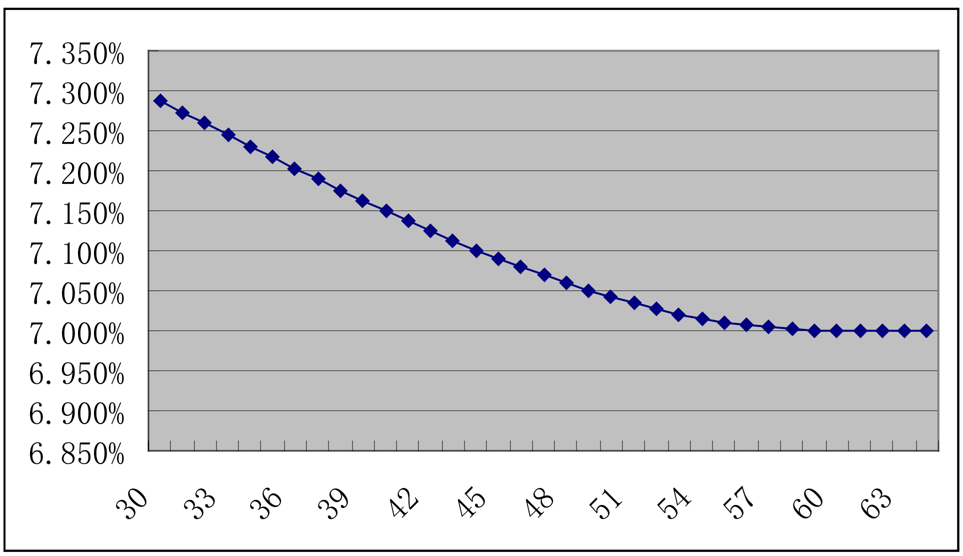
Compared with the Margrabe Model, the difference is that the Money Purchase Account will receive contribution ($c \times S_t$) continuously, which we can view as negative continuous dividend. It is clear that the ratio of cS_t/F_t is strictly decreasing, with range $(0,1]$. We can rewrite equations (9) and (10) into partial differential equations, but it is difficult to find the close form solution for the partial differential equations.

In another way, suppose one member has n-years service, we discretize the n-years service into n periods of 1-year of service. We can project to the normal retirement age with stochastic salary increase and investment return. The advantage of discretizing is that we reduce equation (10) to

$$\frac{dF_t}{F_t} = \frac{dr_t}{r_t} = \mu_2 dt + \sigma_2 dz_{2t} \qquad (11)$$

Then we can apply the Margrabe Model to get the projected option price for 1-year service. From historical data, we get ρ =0.3, σ_1=1%, σ_2 =7.5%. With other assumptions remaining the same as chapter 2: $\ddot{a}_{65}^{(12)} = 10$, $\alpha = 1.7\%$, $c = 10\%$, $\mu_1 = 2.7\%$, $\mu_2 = 7.5\%$, we show the results in Figure 4.1.

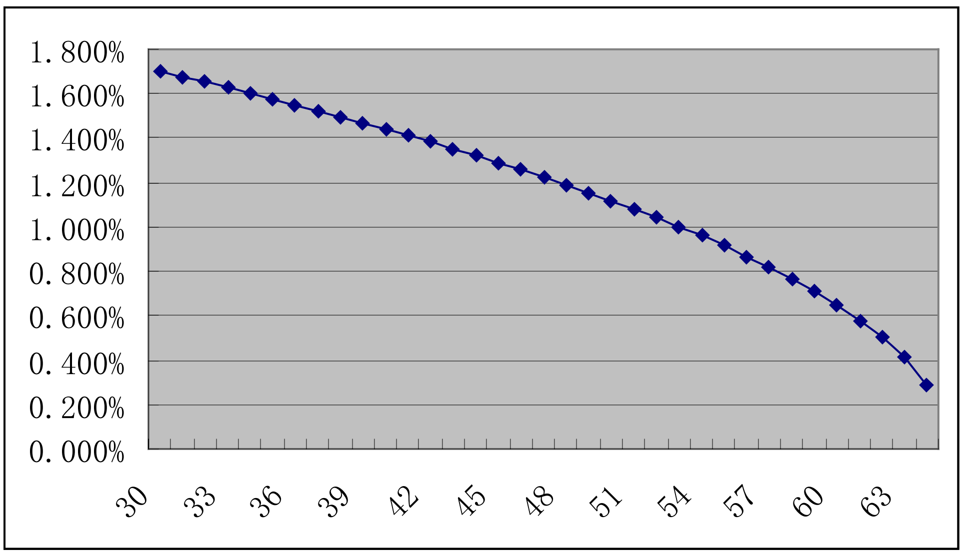
Figure 4.1 Projected Option Price for 1-Year Service as a Percentage of Salary according to Age



From Figure 4.1, we can see that the projected option cost is strictly decreasing from 7.29% to 7% according age. However, the range is narrow, which means the projected option cost for 1-year service does not vary much by age.

The projected option price for 1-year service gives us the extremely high price, since the initial ratio of $G_0\ddot{a}_{65}^{(12)}/F_0$ is 1.7, so that the option is far in the money. Assuming initial funding ratio $G_0\ddot{a}_{65}^{(12)}/F_0$ =1 (i.e. the present value of the minimum guaranteed benefit is equal to the present value of the asset), we get the results in Figure 4.2. It gives us much lower option cost compared with $G_0\ddot{a}_{65}^{(12)}/F_0$ =1.7.

Figure 4.2 $G_0\ddot{a}_{65}^{(12)}/F_0$ **=1**



The results in Figure 4.1 overstate the cost substantially because of 'time-diversification', that is, the excess of the fund accumulation in the time when the option is out of the money is used to offset the cases when the option is in the money. If we allow for reducing money ness of the option, Figure 4.2 may give more realistic option prices.

Another limitation of this exchange option approach is that in this method, both salary growth and investment return rates are modeled in Q-measure, but actually salary is untraded.

Chapter 5
Pricing by Monte Carlo Simulation

The original work on Monte Carlo simulation in option pricing was Phelim Boyle "Options: A Monte Carlo Approach" (1977). Since then Monte Carlo simulation has been used extensively in practice in option pricing.

In previous chapters, we notice that the option cost mainly depends on two variables: the salary increase rate s_i and the investment return rate r_i. Since the two rates are positively correlated, considering the two diffusion processes given in equation (9) and (10), we assume these two series of random variables follow the 2-dimension dependent lognormal distribution with mean vector $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and the variance matrix $\begin{pmatrix} {\sigma_1}^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & {\sigma_2}^2 \end{pmatrix}$.

With other assumptions remaining the same as before, we consider three methods to do the Monte Carlo simulations: stochastic actuarial method, equilibrium pricing model and "risk-neutral" Monte Carlo simulation.

5.1 Stochastic Actuarial Method

In this method, we model both salary and fund in Money Purchase Account in P-measure and assume a passive hedging strategy. Let $V_T = \max\left(G_T \ddot{a}_T^{(12)} - F_T, 0\right)$, and we consider two asset allocations for the additional contributions.

First, assume the additional contributions (option price) are entirely invested in risk free bonds, then

$$V_T = \sum_{t=0}^{T-1} c^* S_t e^{r(T-t)} \qquad (12)$$

where r is the risk free rate, equal to 6%,

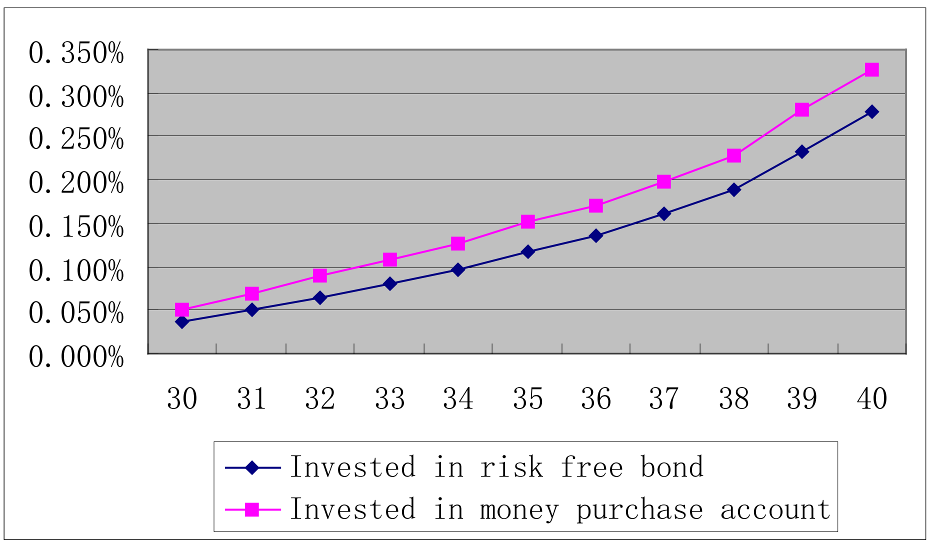
c^* is the additional contribution rate.

Second, assume the additional contributions are entirely invested in Money Purchase Account, then

$$V_T = \sum_{t=0}^{T-1} c^* S_t \exp(\sum_{k=0}^{t} r_i) \qquad (13)$$

Figure 5.1 shows the outcomes.

Figure 5.1 Option Price (as a Percentage of Salary) under Stochastic Actuarial Method



From Figure 5.1, we can see that the option prices under stochastic actuarial method are higher than deterministic actuarial method, which equal to 0 for new entrant age before 40 for all deterministic scenarios in Table 3.1 except the ‘prudent’ scenario. For the Money Purchase fund asset allocation, the additional contribution invested in the Money Purchase Account is higher than the additional contribution following the risk-free bond asset allocation—invested in risk free bonds. This means risk free bonds offer a better static hedge than the Money Purchase Account. This makes sense, because if we put the additional contribution in Money Purchase Account, if the investment return is poor not only will the fund performance be bad, leading to a higher guarantee cost, but also the additional contribution will have a lower value, which will result in an even worse situation.

As with the deterministic actuarial method, the guarantee cost increases according to entry age under this method. If the additional contribution is invested in risk free bonds, the price will reach 0.15% of salary at age 35; if invested in Money Purchase Account, the price will reach 0.15% at age between age 36 and 37. Recall that 0.15% is the current additional contribution. Whether this is sufficient depends on a new entrant pattern. For example, if new entrants enter uniformly at integer age between 30 and 40, the current additional cost under the risk free bond asset allocation would be approximately 0.17%.

5.2 Equilibrium Pricing Model

In modern finance, the two basic approaches to the option pricing are the no-arbitrage theorem and the equilibrium pricing model. In equilibrium pricing model, the equilibrium prices result from the optimizing actions of all the agents in the market. Equilibrium is reached when the prices are such that each agent's expected utility is maximized.

For an option payoff V_T , the equilibrium price is (Panjer 1998):

$$p = \frac{E^P\left[u'(C_1)V_T\right]}{E^P\left[u'(C_1)\right]}e^{-rT} \qquad (14)$$

where

C_1 is a random consumption variable and

$u(x)$ is the power utility function with

$u(x)$ satisfying $u(x) = x^\beta / \beta$, where $\beta < 1$

$$u'(x) = x^{\beta-1} < 0$$

$$u''(x) = (\beta - 1)x^{\beta-2} > 0$$

The utility function is an increasing and concave function. Equilibrium theory can be used to derive the Black-Scholes equation. An advantage of the method is that all expectations use the real world, or P-measure.

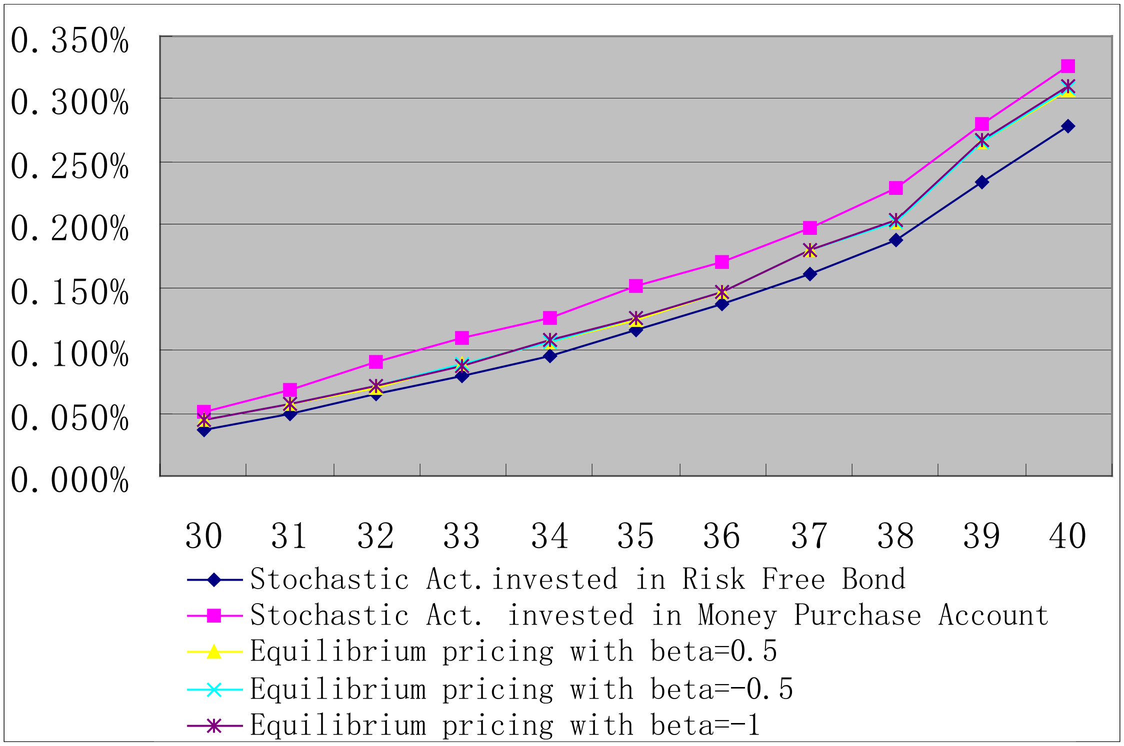
Let salary represent consumption, we use utility function to adjust the option price. Then, from equation (14), we get:

$$p = \frac{E^P\left[u'(S_T)V_T\right]}{E^P\left[u'(S_T)\right]}e^{-rT} \qquad (15)$$

To test whether the option price is sensitive to β , we try $\beta = 0.5$, -0.5, -1.

The results are shown in Figure 5.2. The equilibrium prices are just between the two prices under the stochastic actuarial method. And when we vary $\beta = 0.5$, -0.5, -1, the equilibrium prices almost converge to a single line, which means the equilibrium price has little sensitivity to β .

Figure 5.2 Equilibrium Price



5.3 "Risk-Neutral" Monte Carlo Simulation

In section 1, we simulate the option price under the stochastic actuarial method, assuming both salary and fund are in P-measure; in section 2, we simulate under equilibrium pricing model, where salary and fund are also in P-measure but utility weighted. Now we consider the case when fund is in Q-measure while salary is still in P-measure. This may be justifiable, since salary is untraded, but it clearly means that the model is incomplete and significant basis risk remains.

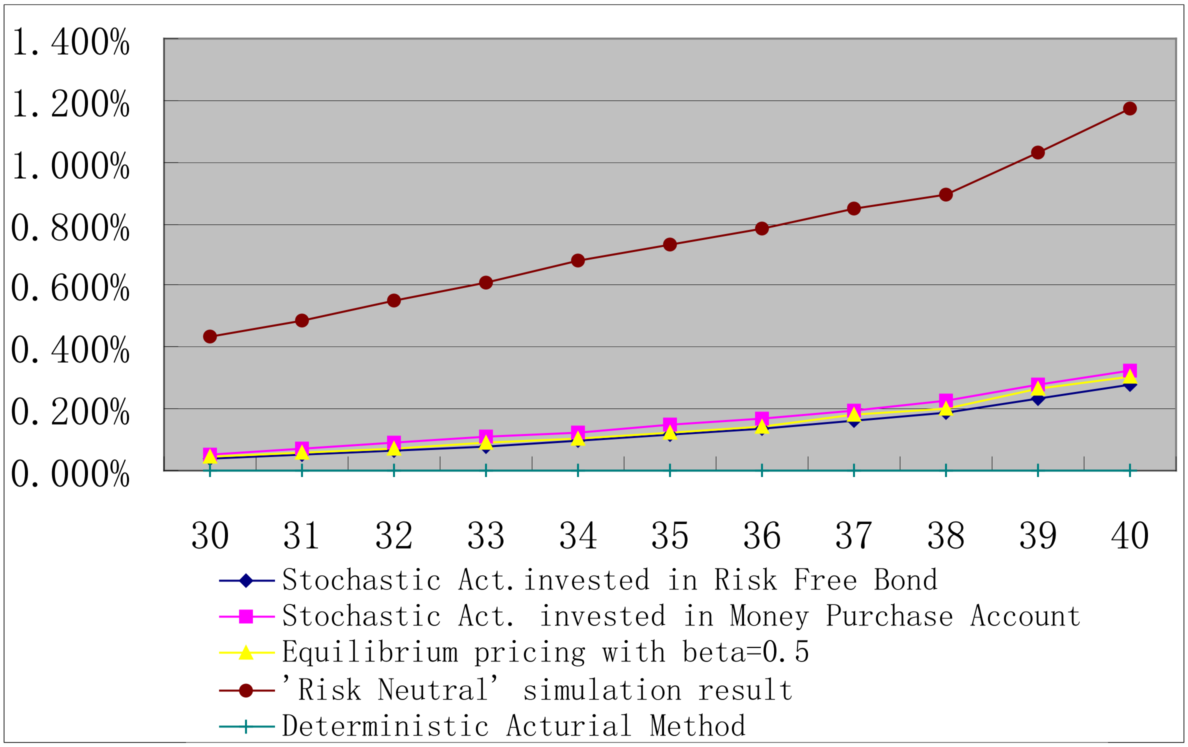
When the fund is modeled in the natural risk-neutral measure, we have

$$r_t \sim \log norm(r_{free} - \frac{\sigma_{free}^2}{2}, \sigma_{free}^2) \qquad (16)$$

where r_{free} =6%, σ_{free} =7.5%

Figure 5.3 shows the results:

Figure 5.3 “Risk-Neutral” Simulation Results



The option price under “risk-neutral” simulation is much higher than the option price under other approaches. The reason is that the fund in this method is in risk neutral measure, which provides a better hedging strategy than other approaches, while the investment return in other approaches is in real world measure. So the option price under “risk-neutral” simulation is costly. For this method, the current additional contribution rate is definitely not sufficient.

Chapter 6
Summary

1. The deterministic actuarial method is not suitable for valuing the cost of the hybrid pension plan. The reasons are given in Chapter 3; the deterministic actuarial method ignores the volatilities of salary growth rate and investment return rate, and so understates the option price. In addition, choosing scenarios under deterministic actuarial method is arbitrary.

2. Whether the current additional contribution rate (0.15%) is sufficient depends on the entry age. Only under the deterministic actuarial method does it seem sufficient. However this method understates the option price. Under the stochastic actuarial method and equilibrium pricing model, the option price reaches 0.15% of salary at about age 35 to 37. Whether it is sufficient depends on a new entrant pattern. Under “risk-neutral” simulation, it is definitely not sufficient.

3. Several studies have explored hedging strategies for guaranteed annuity options, including Boyle and Hardy (2003), Hardy (2000). As shown in Chapter 4, risk free bonds offer a better static hedge for the additional contribution. Inflation-linked bonds may also be a good choice.

4. We have assumed no early exits and priced the put option as European type throughout this paper. In fact, due to death, withdrawal or early retirement, the option may behave as

American type. On the other hand, in reality, the hybrid pension plan has strict regulations on early exits. For example, York University's regulations on early retirement are: for death and withdrawal, the member can only get the fund in Money Purchase Account; for early retirement, if one retires before 60, the guaranteed minimum benefit for the person will have 0.6% deduction per year, if retiring between 60 and 65, the guarantee for the person will have 0.3% deduction per year. All of these regulations will make the cost of option lower. Therefore, ignoring early exits provides an implicit margin for the additional contribution rate.

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