Exam ALTAM

Date: Tuesday, April 23, 2024

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.

2. Question 1 is to be answered in the Excel workbook. For this question, only the work in the Excel workbook will be graded.

3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions, graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations, referencing tables, or for statistical functions, but any work in the Excel booklet will not be graded.

4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions
1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Excel questions will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. You may use any part of the Excel Question worksheet for additional calculations.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.

2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.

3. Write in pen on the lined side of the answer sheet.

4. The answer should be confined to the question as set.

5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.

6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support will not receive full credit.

7. When you finish, hand in all your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

> ***Provide the response for Question 1 in the Excel Question worksheet***

1.

(*10 points*) An insurer issues a 20-year deferred whole life annuity policy to (40). A level premium of 4,000 is paid annually at the beginning of each year during the deferral period. If the policyholder dies during the deferral period, the premiums are returned without interest at the end of the year of death. After the deferral period on survival to age 60, an annuity payment of 10,000 is paid annually at the beginning of each year until death.

Your task is to conduct profit testing using the following assumptions:

      (i)   Mortality follows the Standard Ultimate Mortality Model.

      (ii)   Premium expenses are 4% of each premium.

      (iii)   The earned interest rate is 5% per year.

      (iv)   During the first 5 years of the deferral period, 1% of the in-force policies will lapse at the end of each year. There are no lapses after this.

      (v)   Maintenance expenses are 50 at the beginning of the first year and increase at an inflation rate of 3% per year.

      (vi)   Pre-contract expenses are 500.

      (vii)   The hurdle rate is 10%.

1. Continued

The insurer uses the reserves given below for profit testing the policy.

Time t	Reserve at time t-1	Time t	Reserve at time t-1
0		11	49,680
1	0	12	56,123
2	3,948	13	62,893
3	8,093	14	70,007
4	12,446	15	77,484
5	17,016	16	85,345
6	21,815	17	93,610
7	26,855	18	102,303
8	32,148	19	111,450
9	37,707	20	121,078
10	43,546	21	131,217

(a) (*6 points*)

    (i)   (*3 points*) Calculate the profit vector Pr_t for times $t = 0, 1, 2, \ldots, 20$. You should find that $\mathrm{Pr}_{20} = 11.3$.

    (ii)   (*2 points*) Calculate the profit signature Π_t for times $t = 0, 1, 2, \ldots, 20$.

    (iii)   (*1 point*) Calculate the partial net present value NPV(t) for times $t = 0, 1, 2, \ldots, 20$.
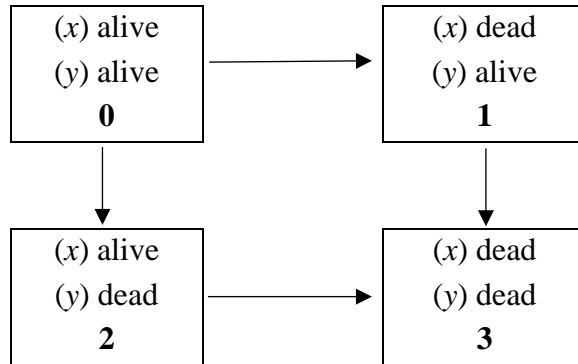
(b) (*3 points*)

    (i)   Explain why the partial NPV is negative for small t.

    (ii)   Determine the Discounted Payback Period.

    (iii)   Calculate the profit margin for the contract based on the cash flows up to the end of the deferral period.

    (iv)   Calculate the internal rate of return (IRR) for the contract based on the cash flows up to the end of the deferral period.

1. Continued

(c)   (*1 point*) The insurer is looking for ways to increase the IRR of the product. Your colleague suggests assuming more policies lapse during the deferral period. Critique their suggestion.

2.

(*9 points*) An insurer models the mortality of couples, denoted (x) and (y), using the Markov multiple-state model described in the following diagram:

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┌─────────────┐              ┌─────────────┐
│  (x) alive  │              │  (x) dead   │
│  (y) alive  │ ───────────▶ │  (y) alive  │
│      0      │              │      1      │
└─────────────┘              └─────────────┘
       │                            │
       ▼                            ▼
┌─────────────┐              ┌─────────────┐
│  (x) alive  │              │  (x) dead   │
│  (y) dead   │ ───────────▶ │  (y) dead   │
│      2      │              │      3      │
└─────────────┘              └─────────────┘
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(a)   (*1 point*) Your colleague states that this model assumes that the future lifetimes of (x) and (y) are independent. State with reasons whether or not your colleague is correct.

(b)   (*3 points*)

   (i)   Write down an integral expression for ${}_t p_{x:y}^{01}$ using multiple state model notation.

   (ii)   Write down an integral expression for ${}_t q_{x:y}^{1}$ using multiple state model notation.

   (iii)   Describe in words the event which has probability ${}_t q_{x:y}^{1} - {}_t p_{x:y}^{01}$.

A couple, (50) and (60), purchase an annuity that pays 10,000 per year continuously while (60) is alive and (50) is dead. The insurer will refund 50% of the net single premium without interest at the moment of death of (60) if (60) dies before (50).

The insurer uses the following assumptions for premiums and policy values:

- The future lifetimes are independent.
- Mortality follows the Standard Ultimate Mortality Model.
- Deaths of (50) and (60) are uniformly distributed over each year of age.
- $i = 0.05$
- $\bar{A}_{x:y} \approx \dfrac{i}{\delta} A_{x:y}$; $\bar{A}_{50:60}^{\ 1} = 0.24898$; $\bar{A}_{55:65}^{\ 2} = 0.06091$

2. **Continued**

(c) (*2 points*) Show that the net single premium is 7250 to the nearest 25. You should calculate the value to the nearest 1.

(d) (*2 points*) Calculate the net premium policy value at the end of 5 years if both lives are still alive.

(e) (*1 point*) At the end of 10 years only the older life, now age 70, is alive and decides to remarry. The survivor requests to apply the policy value as the net single premium for a continuous last survivor life annuity with the new spouse, commencing immediately. State with reasons whether the company would be likely to approve the request.

3.

(*11 points*) NED Life sells special whole life insurances of 100,000 that include a sickness benefit. You are given:

- Pricing and valuation is based on the Standard Sickness-Death Model.

- In the Standard Sickness-Death Model:

$$\mu_x^{01} = a_1 + b_1 e^{c_1 x}; \quad \mu_x^{02} = a_2 + b_2 e^{c_2 x}; \quad \mu_x^{10} = b_1 e^{c_1(110-x)}; \quad \mu_x^{12} = 1.4\mu_x^{02}$$

  where $a_1 = 4 \times 10^{-4}; \ b_1 = 3.47 \times 10^{-6}; \ c_1 = 0.138$
  $\qquad\quad a_2 = 5 \times 10^{-4}; \ b_2 = 7.85 \times 10^{-5}; \ c_2 = 0.087$

- Death benefits are payable at the moment of death.

- Sickness benefits are payable continuously at a rate of 12,000 per year while Sick.

- Level premiums are payable continuously while the insured is Healthy.

- All insureds are age 55 and Healthy at purchase.

- $i = 0.05$

- The net premium rate is 6,622 per year.

- $_t V^{(j)}$ denotes the net premium policy value for a policy in state j at time t.

- $_{10}V^{(0)} = 44{,}250$

(a)   (*1 point*) Show that $_{10}V^{(1)} = 162{,}300$ to the nearest 100. You should calculate the value to the nearest 1.

(b)   (*3 points*)

   (i)   Show that $\left. \frac{d}{dt} {}_t V^{(1)} \right|_{t=10} = -1950$ to the nearest 25. You should calculate the value to the nearest 1.

   (ii)   Using Euler's backward method with a step size of $h = 0.25$, estimate $_{9.75}V^{(1)}$.

3. **Continued**

NED introduces a new product, called Product Q, with the same features as the original policy except that:

- Death benefits are payable at the end of the quarter of death.
- Premiums are payable quarterly if the insured is Healthy at the start of the quarter.
- A sickness benefit of 3,000 is payable at the beginning of each quarter if the insured is Sick then.

You are given that:

(i) $A_{55}^{(4)02} = 0.39127; A_{65}^{(4)02} = 0.53233; A_{65}^{(4)12} = 0.56465$

(ii) $\ddot{a}_{55}^{(4)00} = 10.2478; \ddot{a}_{55}^{(4)01} = 2.3057; \ddot{a}_{65}^{(4)10} = 0.0395; \ddot{a}_{65}^{(4)11} = 8.9373$

(iii) $_{10}V^{(0)} = 43,792$

(c)   (*1 point*) Show that the annualized net premium is 6,520 to the nearest 10. You should calculate the value to the nearest 1.

(d)   (*1 point*) Show that $_{10}V^{(1)}$ for this new policy is 163,450 to the nearest 25. You should calculate the value to the nearest 1.

You are also given:

x	$_{0.25}p_x^{01}$	$_{0.25}p_x^{02}$	$_{0.25}p_x^{10}$	$_{0.25}p_x^{12}$
64.75	0.00674	0.00547	0.00043	0.00764

(e)   (*2 points*) Using backwards recursion with a step size of $h = 0.25$, calculate:

(i) $_{9.75}V^{(0)}$

(ii) $_{9.75}V^{(1)}$

3. **Continued**

NED Life believes policyholders care more about the sickness benefits in the early years and the death benefits thereafter. They introduce a new variation of Product Q, called Product Q+:

- The quarterly net premium for Product Q+ is the same as for Product Q.
- Death and sickness benefits in the first 10 years for Product Q+ are the same as for Product Q.
- No sickness benefits are paid after 10 years under Product Q+.
- The death benefit after 10 years for Product Q+ is increased to F.

(f) (*2 points*) Calculate F.

(g) (*1 point*) The head of marketing proposes that since the premiums and benefits of Product Q and Product Q+ are the same for the first 10 years, any policyholder request to switch between the policies within the 10 years should be granted. State with reasons whether you agree or disagree with the proposal.

4.

(*9 points*) An insurer is analyzing its long term care insurance data. The objective of the study is to estimate transition intensities for the underlying multiple state model illustrated in the following diagram.
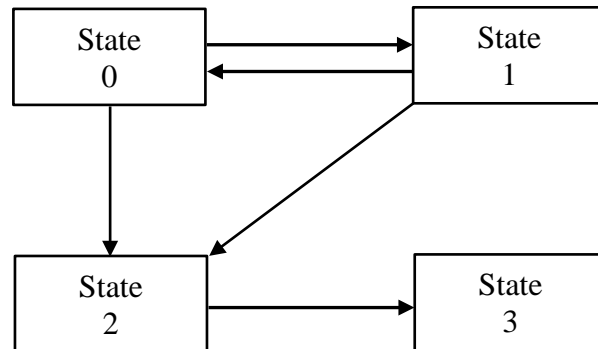
Transition intensities are assumed to be constant over each year of age.

Let $T_x^{(j)}$ denote the total observed waiting time in State j for lives between ages x and $x + 1$, and let D_x^{ij} denote the total number of transitions from State i to State j by lives between ages x and $x + 1$.

(a) (*2 points*) At the start of the two-year observation period, a person age 70.2 is in State 0. At age 71.1 she transitions to State 1, and at age 71.4 she transitions back to State 0. At age 71.9 she transitions to State 2, where she remains until the end of the observation period.

   (i)   Determine the person's contribution to $T_{70}^{(j)}$ and $T_{71}^{(j)}$ for $j = 0,1,2$.

   (ii)   Write down the person's contribution to D_{70}^{ij} and D_{71}^{ij} for $i, j = 0,1,2$, where $i \neq j$.

(b) (*4 points*) For a given age x:

   (i)   Write down the log-likelihood function for estimating the transition intensities μ_x^{ij}, in terms of $T_x^{(j)}$ and D_x^{ij} for $i, j = 0,1,2,3$ and $i \neq j$.

   (ii)   Derive the formula for the maximum likelihood estimator of μ_x^{01}.

4. **Continued**

      (iii)   State with reasons whether the maximum likelihood estimators, $\hat{\mu}_x^{0j}$, are dependent or independent for different j.

      (iv)   Derive a formula for the standard deviation of the maximum likelihood estimator, $\hat{\mu}_x^{01}$.

(c)   (*3 points*) You are given the following aggregated information from the study.

x	$T_x^{(0)}$	$T_x^{(1)}$	$T_x^{(2)}$	D_x^{01}	D_x^{10}	D_x^{02}	D_x^{12}	D_x^{23}
70	3214.0	632.4	317.3	150	42	40	15	20
71	3019.7	640.4	350.7	170	40	55	16	30

      (i)   Show that $\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet}$ is 0.13 to the nearest 0.01. You should calculate the value to the nearest 0.001.

      (ii)   Calculate $_2\hat{p}_{70}^{\overline{00}}$, i.e. the maximum likelihood estimate of $_2p_{70}^{\overline{00}}$.

      (iii)   Calculate an approximate 95% linear confidence interval for $\mu_{70}^{0\bullet} + \mu_{71}^{0\bullet}$.

      (iv)   Calculate an approximate 95% confidence interval for $_2p_{70}^{\overline{00}}$.

5.

(*10 points*) You are the actuary for a small defined benefit pension plan. At the valuation on 1/1/2024 there are 5 members, all of whom retired at age 65 as detailed below. All benefits are paid in the form of a monthly annuity-due payable during the life of the member with payments guaranteed for 10 years following retirement (that is, up to the attainment of age 75).

Member	Exact Age as of 1/1/2024	Annual Pension (000s)
A	69	42.0
B	69	48.0
C	69	60.0
D	72	36.0
E	72	42.0
Total		228.0

The valuation assumptions are as follows:

- Mortality follows the Standard Ultimate Mortality Model.
- Deaths are assumed to be uniformly distributed between integer ages.
- There are no expenses paid from the pension assets.
- $i = 0.05$

(a)   (*1 point*) Show that $\ddot{a}^{(12)}_{72:\overline{3}|} = 10.94$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(b)   (*2 points*) Show that the Total Actuarial Liability (in 000s) for the accrued benefits at the valuation date is 2660 to the nearest 10. You should calculate the value to the nearest 1.

(c)   (*1 point*) Briefly explain why there is no Normal Cost for this plan.

Three years later you are asked to perform a new valuation for the same pension plan. You are given the following information:

- The valuation basis on 1/1/2027 is the same as the basis on 1/1/2024.
- Member B died on 12/31/2026.
- There have been no other changes in membership.
- The plan was 100% funded as of 1/1/2024 (that is, the value of assets in the pension fund was exactly equal to liabilities at that time).
- The plan assets earned a constant rate of return of 3% per year between 1/1/2024 and 1/1/2027.

5. **Continued**

    (d)   (*4 points*)

        (i)   Show that the value of the plan assets (in 000s) on 1/1/2027 is 2190 to the nearest 10. You should calculate the value to the nearest 1.

        (ii)   Show that the Total Actuarial Liability (in 000s) for all members' accrued benefits on 1/1/2027 is 2020 to the nearest 10. You should calculate the value to the nearest 1.

        (iii)   Calculate the aggregate gain or loss on the plan between 1/1/2024 and 1/1/2027.

        (iv)   State with reasons whether the mortality experience of the plan generated a gain or loss between 1/1/2024 and 1/1/2027.

    (e)   (*2 points*) The plan sponsor is considering purchasing annuities to provide the remaining benefits to the retired members. Briefly describe two advantages and two disadvantages to the sponsor of buying annuities at this time.

6.

(*11 points*) A policyholder age 70 purchases two policies:

- A Type B Universal Life (UL) policy with an additional death benefit of 100,000.
- A 10-year equity-linked (EL) insurance policy with a guaranteed minimum death benefit equal to the value of the premiums accumulated at 5% interest, and a guaranteed minimum maturity benefit of the total premiums paid.

You are given the following information about the policies.

UL policy information:

- The policyholder pays premiums of 20,000 at the start of each year.
- The death benefits are paid at the end of the year of death.
- The policy uses a COI rate of 150% of the Standard Ultimate Mortality Model, and an interest rate $i_q = 0\%$ for calculating the cost of insurance.
- Expense charges of 1.5% of the account value (before the premium) plus 2% of the premium are payable at the start of each year.
- There is a level corridor factor of 1.5.
- Each year the insurer sets an anticipated rate that it expects to earn on its EL funds. The credited rate for the UL policy is set at 65% of this anticipated rate, with a minimum credited rate of 2.5%.
- Account values are calculated annually.

EL policy information:

- The policyholder pays premiums of 20,000 at the start of each year.
- The death benefits are paid at the end of the year of death.
- Management charges of 2% of the fund value (before the premium) plus 2% of the premium are payable at the start of each year.
- An additional management charge of 4,000 is payable at the start of the first year.

(a)   (*2 points*) The anticipated rate earned and actual rate earned on the EL funds in the first year are both 9%.

    (i)   Calculate the account value at time 1 for the UL policy.

    (ii)   Calculate the fund value at time 1 for the EL policy.

6.   **Continued**

The policyholder survives to time 9. You are given:

- The account value for the UL policy at time 9 before the premium payment is 196,840.
- The fund value for the EL policy at time 9 before the premium payment is 212,000.
- The anticipated rate and the actual rate earned on the EL funds in the 10th year are both 3%.

(b)   (*3 points*)

(i)   Show that the account value at time 10 for the UL policy is 214,000 to the nearest 100. You should calculate the value to the nearest 1.

(ii)   Show that the fund value at time 10 for the EL policy is 234,200 to the nearest 100. You should calculate the value to the nearest 1.

(c)   (*2 points*) Calculate the payout from each policy if the policyholder dies during the 10th year.

(d)   (*2 points*) Describe two advantages and two disadvantages of the UL policy compared with the EL policy, from the perspective of the policyholder.

(e)   (*2 points*) The policyholder decides to annuitize the EL proceeds at policy maturity (calculated in (b)(ii)). The policy has a Guaranteed Minimum Income Benefit (GMIB) option, allowing the policyholder to annuitize the Benefit Base amount at time 10, at a guaranteed annuitization rate of 11.2% for a life age 80. The Benefit Base is the value of the premiums accumulated at 5% interest.

(i)   Calculate the annual annuity payout under the GMIB.

(ii)   Explain why the policyholder might choose to annuitize the policy proceeds without exercising the GMIB.

END OF EXAMINATION