COMPREHENSIVE SYLLABUS CONSTRUCTION PRINCIPLES WITH ALL COGNITIVE LEVELS

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ABSTRACT

We present comprehensive syllabus construction principles. These principles facilitate a goal of the Bloom pedagogic hierarchy: presenting the full span of cognitive levels needed by the student for mastery, from memorization to higher cognitive levels. The paper also emphasizes a recent advance in pedagogy theory: A reformulation of higher-cognitive level as equated with use of executive function, the brain function allowing simultaneous integration of several brain functions. Two important examples are presented: i) An SOA preliminary exam syllabus module is presented and then significantly improved; ii) Marzano's nine high-yield strategies are presented and also analyzed using executive function.

I: OVERVIEW AND GOALS

The three goals of this paper are to:

- Propose improvements on published, preliminary-exam, SOA syllabi
- Justify these proposed improvements by identifying general syllabus-construction principles which can and should be applied to any subject
- Summarize some simple methods by which to enhance the presence of higher-cognitive thinking in course instruction.

Throughout the first few sections of this paper, we will focus on one illustrative module, the annuity model from the SOA FM syllabus [28]. Figure 1a presents the current full text version of this module. Figure 1b presents the proposed modification. Figure 1c provides a side-by-side comparison of Figures 1a and 1b. It should be obvious that in some sense the Figure-1b syllabus *improves* on the Figure-1a syllabus. Section IV will present four general syllabus construction principles which naturally generate the Figure-1b syllabus version.

Learning Objective: The Candidate will be able to calculate *present value, current value, and accumulated value* for sequences of non-contingent payments.

Learning Outcomes: The Candidate will be able to:
a) Define and recognize the definitions of the following terms: *annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity.*

b) For each of the following types of annuity/cash flows, given sufficient information of *immediate or due, present value, future value, current value, interest rate, payment amount, and term of annuity*, calculate any remaining item.

- o Level annuity, finite term.
- o Level perpetuity.
- o Non-level annuities/cash flows.
  - Arithmetic progression, finite term and perpetuity.
  - Geometric progression, finite term and perpetuity.
  - Other non-level annuities/cash flows.

Figure 1a: Current SOA, Annuity, February-2019, Syllabus Module [28]

Figure 1b presents a proposed modification of this current syllabus module.

The candidate will know:

a) <u>Modalities</u>: *Cashflow timelines, timevalue calculator lines, formulae, and verbal descriptions* for
b) <u>Basic units</u>: The four basic annuity building blocks *(i) level, ii) increasing arithmetic, iii) decreasing arithmetic, iv) geometric (inflation) (increasing, decreasing)* as possibly modified by
c) <u>Basic modifications</u>: The five basic manipulation methods of stand-alone basic annuities *(i) deferral, ii) immediate-due, iii) interest-payment period conversions including continuous payments and interest, iv) finite and infinite payment schema, and v) present-value vs. accumulation calculations* as well as
d) <u>Complex designs</u>: i) Sequential combinations of annuity types (for example, *level and then increasing*, *inflation and then level*) ii) Nested combinations of annuity types (for example, *constant within year and increasing by year*) as well as
e) <u>Basic Industry Applications</u>: *Endowments, retirement, college-funding, …*

<u>Figure 1b</u>: A proposed modification of the annuity module of the SOA FM syllabus

Figure 1c summarizes the contrast of Figure 1a and Figure 1b using a side-by-side format.

Figure 1a: Current SOA Syllabus module	Figure 1b: Proposed modification
Be able to define and recognize… Be able to calculate … • Level annuity (finite) • Level perpetuity • Non-level annuities o -Arithmetic o Geometric o Other	* Know four modalities (*formal, geometric, …*) *Know four basic annuity types (*level…*) * Know five basic annuity modifications (*deferral…*) * Know sequential and nested annuity design * Be familiar with standard industry applications

II: PREREQUISITES: EXECUTIVE FUNCTION AND TEACHING TO THE TEST

Section IV presents the syllabus construction principles which naturally suggest the syllabus version presented in Figure 1b. Before presenting the criteria for challenging syllabus construction we require three small prerequisites. We require understanding and working knowledge of: executive function (Section IIA), teaching to the test (Section IIB), and pedagogic challenge (Section III).

IIA. Executive Function: We will speak about cognitive or pedagogic challenge in this paper. The psychological brain function most connected with cognitive challenge is executive function. Executive function refers to brain activity that integrates several brain areas [22, 31]. Although many features are connected with executive function we find it most useful to concentrate on the aspect of executive function that refers to the simultaneity of integration of several brain areas.

To emphasize this aspect of executive function we use Figures 2a and 2b which are a miniature model of one test for executive function, the Trail-Making test [5, 7, 10, 27].

Figure 2a – The "A" Test		Figure 2b – The "B" Test
1 3 5 2 4		1 3 2 B A

To administer the Trail-Making test, two cards are presented to the examinee. The first card typically has 25 numbers, 1 through 25, scattered randomly. In Figure 2a we have 1 through 5. The examinee must create a trail joining 1 to 2 and then to 3 and then to 4 etc. until all numbers have been sequentially connected. There is no right or wrong answer to this test. But the time it takes to complete the task is recorded. This is known as the "A" test.

The examinee is then given a 2nd card which has 1, A, 2, B, 3, C…. The traditional Part B test has 25 items. In the example above, we have included only five items in sequence. The examinee must connect 1 to A and then connect A to 2 and then to B and then connect B to 3 etc. till all letters and numbers are connected. This is known as the "B" test.

Again, there is no right or wrong answer to this test. But the time it takes to complete the test is recorded. What the examiner will study is the difference in time for completion of Test B and Test A. Several basic results in the literature are as follows:

- Both tests are rather trivial; they are trivial in the sense that most people can easily count 1 through 25 (or 5) and most people know the alphabet A through Z well.
- All people take longer to complete the B test then the A test
- The B test requires using two parts of the brain, the part dealing with numbers and the part dealing with letters. Contrastively, the A test just requires using one part of the mind. It follows that the B test uses executive function since two parts of the brain are being used while the A test does not [10].
- Despite the simplicity of the tests, the difference in time between the A and B test can be used to diagnose brain problems. An excessive difference between the completion time of the B and A tests suggests brain damage. A small difference between the A and B test correlates with a quick recovery time from a stroke [27].

In this paper, we are interested in pedagogic and cognitive challenge. What is important for us, is that pedagogic challenge can be defined and equated with executive function which in turn can be equated with using multiple areas of the brain. Startingly, just using two well-known brain areas (such as recognition of numbers and letters) *is sufficient to create a cognitive challenge that is measurable and meaningful.*

With an eye to the rest of the paper we mention the Huges-Hallett approach to Calculus reform [13]. Hughes-Hallett addressed the problem of teaching calculus in light of the national failure rate. Her remedy to calculus teaching was to make calculus challenging and this in turn was to be accomplished by the rule of four, the teaching of calculus examples and exercises using the four brain areas of i) verbal, ii) formal, iii) geometric, and iv) computational. If, for example, you wanted to teach finding the maxima of a function, it is not enough to show how to differentiate, set the derivative to 0, and perform a further test to see if this zero-point is a maxima; rather, the serious student must also understand maxima in terms of graphs and in terms of computational tables. We will elaborate further in Section II.C below.

But it immediately follows that Hughes-Hallett is advocating remedying the national failure rate in teaching calculus by incorporating executive function, the use of multiple brain areas.

II.B Teaching to the Test: There is a rich literature on teaching a course, teaching to the syllabus, teaching to the test, as well as the integration of proper testing with syllabus construction [6, 16, 17, 18, 19, 21, 23, 24, 25]. This is especially relevant to courses designed to cover the SOA syllabus for preliminary exams.

A crucial question is the relation between the syllabus and the SOA exam material. The following bullet points are critical:

- Teaching to the test is only bad if one *restricts* the syllabus. For example, if the instructor says your exam will come from problems 1-60 in a certain section, the instructor has restricted the syllabus; the student need not study all material, rather, it suffices to know how to do problems 1-60.
- Teaching to the test is good if one *expands* the syllabus.
    - An example relevant to this paper is the following: The instructor informs students that the test will consist of problems where either i) comparisons will be made between two actuarially equivalent annuities (e.g. find a level annuity equivalent to an inflation adjusted annuity) or ii) annuities needed to be analyzed as a sequential combination of several basic annuities (e.g. an increasing annuity that levels off after a few payments)

    - Such a teaching to the test is good because it *expands* what the student must know. In fact, a student studying from the SOA syllabus, Figure 1a, reading this example, would expand the skills they must learn to include multi-component problems. Such an *expansion* of requirements is good since it pushes the student to study beyond the normal requirements.

- The exam material and syllabus should intertwine such that each informs the other. More specifically, if an instructor knows in advance what types of problems students must be able to answer, then that advanced knowledge of problem types can inform the syllabus allowing codification of those problem types into the syllabus [24].

  In fact, a major driving force behind the construction of Figure 1b, was such an informing of the syllabus by SOA-selected, model, exam questions. More specifically, a review of SOA-selected, model, exam questions [29] revealed that many questions deal with sequential and nested annuities. This motivated adding the sequential and nested annuity bullet to Figure 1b. Such explicitization of problem types in the syllabus is beneficial for the candidates since they are encouraged to focus on the skill of problem analysis into component parts (that is, sequentiality and nesting of annuities) needed to solve these problems.

  A possible objection to such explicitization of the syllabus informed by exam problem prototypes is that it constricts the instructor who is now bound exclusively by the problem types listed in the syllabus. This objection is easily answered: The very goal of the syllabus is not only to define the content the candidate needs to master but to articulate its boundaries. The syllabus should give an exhaustive account of what skill sets lead to content mastery. The skill sets should include both content-specific items about annuities as well as needed cognitive skills such as methods of combining problems. A careful review of several hundred SOA-selected, model, exam questions [29] shows that the syllabus presented in Figure 1b explicitly covers each such problem.

- Here is still another way to understand the Figure 1b-syllabus. Figure 1b is claiming that a candidate who has mastered the following skills is proficient in annuities.
    - The candidate knows how to price (present values) and calculate accumulated values for any series of periodically recurring payments subject to the following forces:
        - The payments naturally divide into consecutive or nested groups of payments with

- Each group of successive payments of portions of the annuity either level, arithmetically increasing, arithmetically decreasing, or geometrically increasing/decreasing with
- The payments varying in i) begin or end-period, ii) alignment or variance between payments and interest period calculations, iii) finite-infinite, iv) deferrals, v) present-value vs. accumulated value computations.

The reader is challenged to identify anything left out. One way of assessing this syllabus is against published lists of available problems and such an assessment is positive; all topics informed by the model problems are accounted for.

To recap the theme of this section, a syllabus informed by a large universe of model problems, is an expanded syllabus. Such an expansion benefits both instruction and exam assessment by providing transparency of requirements. Such an expansion is not teaching to the test in the pejorative sense of the word since it does not *restrict* required candidate studying, but on the contrary *expands* it. Similarly, such an expansion does not restrict the exam item writer but on the contrary empowers the exam item writer by explicitly identifying the types of complexity they can and should address.

III PEDAGOGIC CHALLENGE

In this section we review three sources of definition of pedagogic challenge: A) the pedagogic hierarchies of the 20th century, B) Hughes-Hallett's calculus reform, and C) Hendel's executive-function approach.

III.A The Pedagogic Hierarchies: Abraham Bloom [4] created the first pedagogic hierarchy which was later modified by Anderson [1]. The idea of a hierarchy is that early levels in the hierarchy indicate cognitively simple tasks while later levels in the hierarchy indicate cognitively challenging tasks. The six Bloom-Anderson levels, with illustrative examples, are listed in Figure 3a. Since Bloom, many alternate hierarchies have been presented including those of Gagne [9], Marzano [20], and Van-Hiele [32]. The Marzano hierarchy is presented with illustrative examples in Figure 3b. Recent research points to the equivalence of efficacy of these hierarchies [35].

Figure 3a: Bloom-Anderson		Figure 3b: Marzano
Memorize– *list, define, know, tell*		*Recognize-recall*
Identify – *cite, describe, outline, ask*		*Represent-symbolize*
Apply – *organize, use, illustrate act,*		*Analyze*
Analyze – *examine, dissect, investigate, order*		*Problem – Decision Making*
Synthesize– *design, produce, imagine, invent*		
Evaluate – *compare, critique, recommend, test*		

III.B Hughes-Hallett. Unlike Bloom, Anderson, and Marzano, Deborah Hughes-Hallett does not specialize in psychology or instructional design. Rather, Hughes-Hallett introduced calculus reform [13]. Hughes-Hallett's basic idea is that calculus should be taught with what she called the *rule of four*, that is, every idea should be presented using four modalities: verbal, graphical, formal-algebraic, and computational.

This approach is summarized in Figure 3c using extrema as an illustrative example. Every calculus student knows that extrema can be located using the 1st and 2nd derivative tests, algebraic manipulations of the underlying functions. Hughes-Hallett argues that this is not enough. Students should be able to identify extrema using graphs (e.g. the simple idea that the vertex of a V-like graph is the minima of that graph and its underlying function). Students should be able to identify extrema using function tables. Finally, students should be able to identify a request for minima in a verbal problem.

RULE OF FOUR
Formal -Algebraic – *locate extrema using the 1st and 2nd derivative tests*
Graphical – *locate extrema using graphs and visual identification of points*
Verbal – *identify requests for extrema in verbal problems*
Computational – *identify extrema through a function table*

Figure 3c: Debra Hughes-Hallett's Rule of Four.

Although, Hughes-Hallett's ideas are specific to calculus, I have advocated generalizing them to a definition of pedagogic challenge: Pedagogic challenge is enhanced if multiple modalities are used in teaching and assessing the concept [11].

III.C Executive Function: Hendel, in a critical review of half a dozen pedagogic hierarchies as well as the Hughes-Hallett approach to calculus, identified an underlying unity [12]. Psychology teaches that executive function refers to the brain function that allows integration of several brain areas. Thus, executive function is the unifying feature to Hughes-Hallett's rule of four, as well as the *analysis-synthesis* levels of Bloom-Anderson, or the *knowledge utilization* of Marzano. Any cognitive activity that uses multiple areas, whether assessing the multiple parts of a procedure (Bloom's *analysis* level), creating an innovative design involving integrating a specific learned skill with another area (Bloom's *synthesis* level), or applying a learned skill to a new situation (Marzano's *utilization* level), as long as multiple brain areas are used, is employing executive function. Hendel therefore argues that the true driver of higher cognitive thinking is executive function.

To emphasize the power of executive function even in a simple form we re-visit the trail-making test introduced in Section IIA. Despite the simplicity of the two-brain areas (recognizing numbers and letters), the Trail-Making test is a very powerful test that can diagnose brain health and damage.

IV: FOUR SYLLABUS CONSTRUCTION PRINCIPLES FOR SYLLABUS IMPROVEMENT

In this section we present the proposed syllabus construction principles along with illustrative examples from Figures 1a, 1b, 1c. Each principle is accompanied by a discussion of cons and pros.

Principle A: Basic Formulae:

A syllabus should give candidates:

- Information on both:
  - Lower cognitive level skills, such as topics requiring memorization, as well as,
  - Higher cognitive level skills, such as problems which require analysis and synthesis.
- Enumeration or lists of items to memorize for each topic.

This enhances, not detracts, from the overall higher-level nature of the syllabus since it identifies an exhaustive and final list needed for mastery.

Illustrative example: The proposed actuarial-syllabus module (Figure 1b) explicitly identifies four basic annuity building blocks (*level, increasing arithmetic, decreasing arithmetic, and geometric(inflation)).* It also explicitly identifies five methods of modification: *deferral, due-immediate, finite-infinite, payment period vs. interest period, present-value vs. accumulation*. This is intended as an exhaustive list for which candidates should know and memorize all formulae.

Discussion: It might appear "wrong" to indicate a memorization requirement in a syllabus. However, the creators of the pedagogic hierarchies, such as Bloom and Anderson both emphasize that it is wrong to *only* concentrate on higher level thinking. Both instruction, and consequently, the syllabus, should present a balance of rote and higher-level learning.

Here is a critical supportive argument: How can a candidate analyze a complex annuity payment flow into component parts unless the candidate is aware of the basic building blocks and modifications possible? In fact, when a student comes to this author during office hours stuck on a complex problem, their difficulty might be that they are not using analysis (a higher cognitive level skill) and breaking up problems into parts; but their difficulty might also be that they are unaware of the finite list of building blocks from which this annuity is built. In fact, this author has been successful with certain students by asking the simple question, "What are the basic building blocks; which ones do you think applies to this problem?"

Finally, it should be pointed out, that if instructors do not list the material to memorize on their syllabus, then students go elsewhere for 'formula sheets.' This shows that memorization is recognized and needed; consequently, it should be part of the syllabus.

Principle B: Higher Cognitive Thinking: High-level descriptions of problems and situations requiring analysis and synthesis should be indicated.

Illustrative example: The proposed actuarial-syllabus module (Figure 1b) mentions *sequential* and *nested* annuities. An example of a *sequential,* annuity, payment-flow problem would be a request to price a perpetuity that increases $1000 each year for 10 years and then remains level for the remainder. An example of a *nested,* annuity, payment-flow problem would be the request to price an annuity that is level monthly but increasing yearly for 40 years.

Discussion: One concern of syllabus writers is *teaching to the test.* However, the testing literature, emphasizes that *teaching to the test* is bad if the indicated material restricts what the student learns; *teaching to the test* is good and in fact *teaching to the syllabus* if it expands what the student learns [6, 16, 17, 18, 19, 21, 23, 24, 25]. If an instructor indicates that the syllabus (or test) covers specific problems, then the instructor has restricted student learning. Students need only learn those problems. But if the syllabus simply states that a *student should be able to analyze sequential and nested annuities into their component parts and synthesize the results to obtain final payments,* then the instructor has *expanded* what the student must learn. The syllabus focuses the student on certain combination methods thereby expanding what must be learnt, since the student must be prepared to solve *any* annuity with arbitrary sequential and nested parts.

Principle C: Modeling: A syllabus should explicitly include the requirement to be able to model verbal problems on any computational topic. Current, regularly-occurring, industry examples should be included.

Illustrative Example: Standard industry annuities such as college-funding, retirement, and endowments should be included in any syllabus.

Discussion: Admittedly, it is difficult to exhaustively identify all standard industry examples. Each instructor should have several such examples, *to the extent possible,* in his course syllabus. Needless to say, an escape clause such as *Industry examples such as….* should be included to preclude an appearance of exhaustivity.

Principle D: Syllabus-Test alignment: The modern approach to syllabus-test interaction is that the intended test questions should inform the syllabus [25]. A good instructor may have a (mental) test bank of several 100 typical test items for a course. One approach to syllabus construction is to review test items with an eye towards nuances of standard syllabus topics. To use an analogy, the syllabus is like a barren tree with just roots and branches, while the syllabus informed by a test bank of problems with syllabus nuances inferred from model test problems, is a leafed tree.

Illustrative example: The proposed syllabus, Figure 1b, was in fact constructed by reviewing the 200, model, SOA, annuity problems [29]. The review of this problem collection showed certain key features and ideas that have been incorporated into the proposed syllabus which were not in the SOA syllabus (Figure 1a). More specifically, the Figure 1b syllabus introduces the concept of nested annuity and also explicitly identifies the lists of four basic annuity types, five basic annuity modifications, and two higher level cognitive methods (sequential and nested); these concepts and lists cover all problems.

While there is always concern that something new can be discovered, and while syllabi should have elastic clauses (e.g. other annuity related topics as they arise), this concern should not deter an instructor from explicitly identifying criteria which cover 99% of the cases. This is a crucial point. Students given an atmosphere of completeness, all they need to know, or more motivated to achieve mastery. Contrastively, informing them that ‘other material’ may be used, is negative and inhibits motivation.

V: APPLICATIONS OF EXECUTIVE FUNCTION TO ACTUARIAL TEACHING

While this paper has emphasized memorization as well as higher-order cognitive skills, the primary emphasis is on higher-order cognitive skills. Marzano, in his research, emphasizes nine high-yield strategies for instructors [2] which are summarized in Table 5a, along with an executive function interpretation of these methods.

The narrative following Table 5a further explores these ideas. The idea here, is that besides the syllabus, instruction itself very often benefits from executive function.

In the remainder of this section, we review each of the five items in the Table 5a with an eye of emphasizing the executive function in them and how they can be used in a classroom setting. Rather than present the five items in sequential order, we present them in the order with which they most naturally use executive function.

V.A Cues – Organizers: Some instructors, particularly college instructors, might recoil at the idea that teaching cues and organizers are the responsibility of the instructor. However, the mathematical curriculum does use many well-known cues and organizers; these occur in textbooks, instructors use them, and there is no reason they shouldn’t be listed (as one-word hints) in syllabi. Here are some examples of well-known cues and organizers:

Marzano's Nine High Yield Methods	Executive Function Application	Why executive function is involved? Which two areas of the brain are addressed?
1. Similarities and contrasts	Tables	The rows and columns represent multiple dimensions by which each table cell is examined.
2. Summarizing	Gist	The high-level summary of a problem (the gist) and the detailed approach involve different areas of the brain (intuitive for high level and technical for detail level)
5. Non-Linguistic	Graphical, computational,	Rule of four (graphical, computational, formal, verbal)
8. Hypothesis testing	Tree methods	Tree methods (Considers multiple branches of item)
9. Cues – Organizers	Puns, mnemonics	Form (spelling) and meaning

Table 5a: Application of five of Marzano's high-yield methods to classroom instruction and their use of executive function. Methods #3,6,7 of Marzano's nine high-yield methods deal with social aspects of learning (recognition of student success, cooperative learning, and feedback from instructor) while method #4 deals with opportunities for practice. These four methods do not involve executive function.

- SOHCAHTOA: This is the mnemonic that relates three trigonometric functions - sin, cosine, and tangent – to the parts of the triangle whose ratio equals the function. For example, *s*in = *o*pposite/*h*ypotenuse. Wolfram Mathworld, a high-level computer and mathematics website, does not hesitate to dignify one of its pages with this mnemonic [34].
- FOIL: This is the mnemonic for factoring quadratics polynomials into two binomials. It indicates that the *f*irst, *o*uter, *i*nner, and *l*ast factors in the two binomial factors must match in a specified way the original quadratic polynomial [3, 33]. Khan academy, a respectable online curriculum, does not hesitate to dignify this mnemonic in one of its lessons [14].
- QUOTIENT RULE: The (silly) sing-song mnemonic "Lo D-Hi minus Hi D-Lo over the square of what's beLO" helps students remember the quotient rule. This mnemonic is explicitly mentioned in online calculus material for several college sites [15, 26].

The executive function nature of a mnemonic, the relationship between word form and word meaning, occurs in a several-decade old executive function test, the Stroop test [30]. This test observes that there are more errors and it takes longer for people to identify the color font of words when the meaning of the word contradicts its meaning. For example, it takes longer to recognize the font-color of several dozen words like WHITE if the color of the font, in this case, black, contradicts the meaning of the word, in this case white. The reason for greater error and longer time is because two areas of the brain, one for font form and one for word meaning, are being used.

In conclusion, we consider cues and organizers not to be silly but rather high-level. They are cognitively challenging because they meet the definition of cognitive challenge presented in this paper, executive function. We have cited several public sources that are not embarrassed to use such mnemonics in upper level courses. We therefore urge instructors to use such mnemonics.

In this spirit, my favorite mnemonical aid for FM is the observation that an annuity *d*ue has two *d*ots on top of it. The proposed syllabus in Figure 1b, can easily be enhanced with the mnemonic, ILIAD, for the four basic annuity types (*I*nflation, *L*evel, *I*ncreasing *A*rithmetic, *D*ecreasing arithmetic).

V.B NON-LINGUISTIC: We have already discussed the "rule of 4" in Sections II and III. This rule requires that mathematics should be approached through four modalities: formal, geometric, computational, verbal.

Here are some simple examples that the author uses when teaching Financial Mathematics:

- *All* course problems, whether homework or class illustrations, are *always* verbal. The class is never shown a purely mathematical problem divorced from a real-world setting indicated by a verbal problem.
- To the extent possible, derivations are geometric vs. algebraic. For example, the fact that an *n*-year annuity due is the sum of 1 and an (*n*-1) annuity immediate is elegantly illustrated by the cashflow timelines in Figure 5b. No formulae are need. This cashflow timeline approach can be extended to several more sophisticated formulae

```
Timeline:         0------1--------2------3-------n-1-------n
Annuity due:      1      1        1      1    …    1

Payment of 1:     1
Annuity immediate:       1        1      1 …       1
```

  Figure 5b: The present value of an *n*-year annuity due equals the sum of 1 and an (*n*-1) year annuity immediate.
- Most refinancing problems can elegantly be done by skillful use of a few calculator keystrokes. This is vastly superior to formulaic solutions. Figure 5c computes the new payments resulting from a refinancing of a 30-year monthly payment loan of 100,000 at 6% payable monthly, to a 10-year monthly-payment loan starting at t=10, at an interest rate of 15% payable monthly.

N	I	PV	PMT	FV	Comment
12*30	½	-100,000	CPT=599.55	0	Monthly payment
12*20	keep	CPT=-83,686	Keep	0	Outstanding Loan Balance
12*10	1.25	Keep	CPT=1350	0	New monthly payment

  Figure 5c: Refinancing of a 30-year, monthly payment loan of 100,000 after 10 years with a 75 basis point rise in the monthly rate and with a reduction of 10 years of payment. The symbols `N, I, PV, PMT, FV,` refer to the Texas Instrument BA II Plus Time Value line.

Instructors should be encouraged to teach with multiple modalities. Superior pedagogy is achieved when the instructor shows how solution approaches with certain modalities are significantly more efficient.

V.C: Tables: Tables are intrinsically multi-dimensional in nature, the column headings representing different dimensions. Use of the table medium highlights commonality and difference thus facilitating learning and retention. We illustrate with Table 5d, a brief excerpt from a full money-growth table.

To facilitate understanding the executive function in this syllabus module, money growth, we review the executive function present in the Stroop test, a test of executive function, presented in Section V.A. In the Stroop test, each word such as WHITE is perceived as two-dimensional containing the dimensions of *font-*

color (e.g. black) and *meaning-color* (e.g. white). Although this multiplicity of dimension is simple it nevertheless is clinically measurable: Memorization of word lists show greater error and longer recall times when the two dimensions contradict each other. The executive function in the money-growth table is similar and arises from the fact that each column in the table corresponds to a separate dimension. Examples are now given using "weak-student" errors encountered in tests on money growth functions.

Money Growth Method	$A(0)$	$A(t)$	Parameters	Money Growth Function $A(t)/A(0)$	Discount factor from s to t, $v(s,t)$
Compound interest	$A(0)$	$A(t)$	i	$(1+i)^t$	$1/(1+i)^{t-s}$
Simple interest	$A(0)$	$A(t)$	i	$1+ti$	$(1+si)/(1+ti)$
Compound discount	$A(0)$	$A(t)$	d	$1/(1-d)^t$	$(1-d)^{t-s}$
Simple discount	$A(0)$	$A(t)$	d	$1/(1-dt)$	$(1-dt)/(1-ds)$

Table 5d**:** Excerpt from a money-growth function table. A full table would include nominal rates, force, and irregular growth functions (e.g. quadratic)

- The table form shows five dimensions indicated by the column headings. The first two columns indicate commonality of all rows: All rows are dealing with a single deposit at time $t=0$ and accumulation at time t. However, the rows differ in the parameters they use, as well as the money growth and discount functions. Thus, executive function is needed to *read* the table and *understand* it.
- Compound discount has two *minus* signs in its money-growth function, one in the denominator and one in the exponent. Weaker students tend to omit these signs, showing lack of application of executive function, since these students confuse compound discount with two *minus* signs with compound interest without minus signs.
- Similarly, the simple money growth functions (simple interest and simple discount) do not have the nice distance property (distance of t-s) in their respective discount factors while compound money growth functions do. Again, weaker students then to overlook this distinction because they are not fully applying executive function. Clearly, the tabular form facilitates recognizing this as an issue and encourages initial learning.

We therefore recommend use of tables to highlight simultaneous commonality and differences. This will encourage initial learning of subtleties and facilitates avoiding error.

V.D Hypothesis Testing: Marzano undoubtedly had a broader range of problems in mind when he mentioned hypothesis testing; but we will suffice with illustrating this principle with mathematical problems involving cases. The standard such example is an application of Bayes Theorem; an illustrative problem is presented in Figure 5e.

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A population of one million people insured has 65% adults. 90% of adults have no claims while 70% of teenage drivers have no claims. Calculate the percentage of claims coming from teenagers.
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Figure 5e: A standard Bayes Theorem Problem

Students typically find Bayes' theorem problems confusing. However, considering each case of the problem separately removes some confusion. This is conveniently done with the graphical aid of a tree as illustrated in Figure 5f.

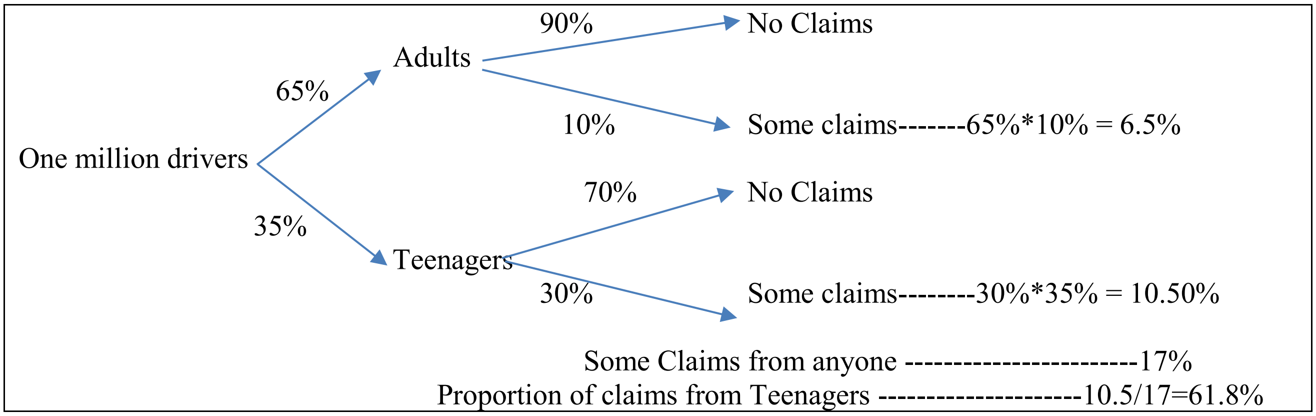

Figure 5f: Use of a probability tree to illustrate flow of logic for the solution in the problem presented in Figure 5e. For further clarification see the text.

Let us carefully examine how the tree facilitates simplifying the problem.

First tree level: At this level we divide the one-million driver population of insureds by age. The problem only states that 65% of the population are adults. But once the question is posed, it is easy to see that the non-adults, say teenage drivers, must constitute 35% (100%-65%) of the population. We can express this in traditional probability symbols:

```
P(Adult) = 65%                                   P(Teenager) = 35%
```

Adult branch: We now focus on the *case* of adults. The tree facilitates this focus since it corresponds to pursuing in depth one branch of the tree. The problem tells us that 90% of adults have no claims. This is shown in the tree. Again, the tree naturally suggests *completing* the children of the *adult* node: So, 10% of adults have no claims.

We could express this with traditional conditional probability symbols:

```
P(No claim | Adult) = 90%;      P(Some claim | Adult) = 10%
```

Leaf path: Recall that a *path* in a tree, is simply a path from the root of the tree (on the far left) to a terminal leaf in the tree (on the far right). The leaf-path rule states that the multiplication of probabilities along the path gives the probability for the compound event obtained by conjuncting the labels along the path. We can summarize this either in English or probability.

```
Probability of Adult AND some claim = 65% * 10% = 6.5%

P (Adult Claim) = P(Adult) P (Some Claim|Adult) = 65% *10% = 6.5%
```

It should be obvious that the probability symbols don't add anything; we don't really need them. There is perfect rigor and complete understanding using the tree. Furthermore, the tree visually separates the various *cases* of the problem facilitating solution.

Problem Completion: The completion of the problem is outlined in the tree and needs no commentary. Using the teenage branch, we compute that

```
Probability of teenager and some claim = 35%* 30% = 10.5%
Probability of any claim (from a teenager or adult) = 10.5% + 6.5% = 17%
Proportion of claims coming from teenagers = 10.5% / 17% = 61.8%
```

The above example illustrates how tree structures facilitate dealing with problems that have two or more cases. Since they have two or more cases they involve executive function and slow down the thinking process. The tree-diagram facilitates identifying separately each case (diluting the executive function). The tree structure is also a viable rigorous alternative to a formal-notational approach using the Bayes Theorem formula.

V.E Summarizing: We use an example from the SOA LTAM exam to illustrate how summarizing involves executive function and leads to improved comprehension. Consider the problem of pricing a premium for a term life insurance with or without features [8]. At least three products come to mind

i. A single premium to purchase a term life insurance policy
ii. A periodic monthly premium to purchase a term life insurance policy
iii. A periodic monthly premium to purchase a term life insurance policy with return of premiums if not death occurs.

These possibilities are summarized in Figure 5g. As shown in Figure 5g, by using summarizing and gisting of the three problems we obtain a unifying principle: INFLOW =OUTFLOW. The summarizing equation INFLOW=OUTFLOW involves executive function for two reasons:

- INFLOW=OUTFLOW is an English sentence and uses a different area of the brain then the part of the brain used for mathematical formulae
- INFLOW=OUTFLOW does not involve technical details like variables and subscripts. This however is not a genuine issue since the translation of term insurance into a mathematical symbol involves a low-level pedagogic cognitive level, memorization of a formula; the student simply has to know the name of the formula, and how to calculate it.

INFLOW=	OUTFLOW
Single Periodic Premiums	Term insurance Accumulated value of periodic premiums

Figure 5g: The figure shows how the verbal high-level equation INFLOW=OUTFLOW unifies a variety of otherwise disparate products: Single or periodic premiums for a term insurance with or without a return of premium feature.

Nevertheless, the summarizing equation achieves something:

- Without the summarizing equation, INFLOW=OUTFLOW, the three products presented in Figure 5g are unrelated. If you learned two of them you would not immediately know the 3rd. For example, if you have been taught how to price using a single premium or monthly premiums, you would be at an (initial) loss to price when there is a return-of-premium feature.

- With the summarizing equation, however, the student is equipped to deal with new situations. If the student, for example, had never seen a return-of-premium feature, they would nevertheless be equipped to deal with it: They would simply have to classify it as INFLOW or OUTFLOW and then mathematically describe it.

In conclusion, *summarizing* and *gist*, involve non-technical, high-level, English statements which unify disparate problem types and facilitate students dealing with new situations.

VI: CONCLUSION

This paper has comprehensively reviewed the key features of syllabus construction with a special emphasis on those features using executive function. As can be seen in Figure 1c, the proposed syllabus design has many superiorities over current syllabi. The following are items highlighted in this paper:

- A syllabus should include both items requiring memorization as well as higher levels of thinking. This is consistent with the pedagogical hierarchies such as the Bloom hierarchy
- Items requiring memorization should be enumerated
- The syllabus should emphasize the rule of four: *Verbal, geometric, computational, formal*
- The syllabus should explicitly include at a high level, executive function topics and the methods that will generate them (such as sequential and nested annuity problems)
- The syllabus and intended test bank of questions should inform each other and be aligned
- We have further reviewed five important executive function pedagogical aids identified by research as facilitating learning:
    - Use of mnemonics (e.g. ILIAD for the four types of basic annuities)
    - Rule of four
    - Use of *gist,* such as a verbal summarizing equation to unify disparate problems
    - Use of tree-structures to deal with multi-case problems
    - Use of tables to clarify multi-dimensional topics.

We believe that consistent application of these principles, as illustrated in this paper, will greatly enhance syllabus clarity and improve both instruction and learning.

VII: REFERENCES

[1] Anderson, L. W., & Krathwohl, D. R. (Eds.). (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's taxonomy of educational objectives*. New York, NY: Longman.

[2] Association for Supervision and Curriculum Development (ASCD), Marzano, Robert J., Pickering, Debra J., & Pollock, Jane E, authors, (2001). *Classroom Instruction that Works: Research-Based Strategies for Increasing Student Achievement,* 1st edition. London, UK: Pearson Publishers.

[3] FOIL Betz, William (1929). *Algebra for Today* (vol. 1). Ginn and Company, p. 291

[4] Bloom, B. S. (1956). *Taxonomy of educational objectives: The classification of educational goals*. New York, NY: Longmans, Green and Company.

[5] Bowie, C. R., & Harvey, P. D. (2006). Administration and interpretation of the Trail Making test. *Nature Protocols*, *1*(5), 2277–2281. doi:10.1038/nprot.2006.390 PMID:17406468

[6] Boyle, B., & Bragg, J. (2009). "How teaching to the test can undermine performance." *Literacy Today*, 58, pp. 25-27.

[7] Corrigan, J. D., & Hinkeldey, N. S. (1987). Relationships between parts A and B of the Trail Making test. *Journal of Clinical Psychology*, *43*(4), 402–409. doi:10.1002/1097 4679(198707)43:4<402: AIDJCLP2270430411> 3.0.CO;2-E PMID:3611374

[8] Dickson, David C. M., Hardy, Mary R., & Waters, Howard R. (2013). *Actuarial Mathematics for Life Contingent Risks (International Series on Actuarial Science)* 2nd Edition. Cambridge, UK: Cambridge University Press

[9] Gagne, R. M. (1985). *The conditions of learning and theory of instruction* (4th ed.). New York, NY: Holt, Rinehart and Winston.

[10] Gaudino, E. A., Geisler, M. W., & Squires, N. K. (1995). Construct validity in the Trail Making Test: What makes Part B harder? *Journal of Clinical and Experimental Neuropsychology*, *17*(4), 529–535. doi:10.1080/01688639508405143 PMID:7593473

[11] Hendel, R. J. (2015a). The rule of four, executive function and neural exercises. *Journal of Systemics*, *Cybernetics, and Informatics*, *13*(5), 14–19.

[12] Hendel, Russell Jay (2017), *Leadership for Improving Student Success through Higher Cognitive Instruction,* in Ronald Styron and Jennifer Styron, (Eds.), *Comprehensive Problem-Solving and Skill Development for Next-Generation Leaders* (pp. 230-254), Dauphin, PA: IGI Publishing, 2017.

[13] Hughes-Hallett, D., McCallum, W. G., Gleason, A. M., Connally, E., Lovelock, D., Patterson, C., & Tucker, T. W. (2013). *Calculus: single and multivariable* (6th ed.). Hoboken, NJ: Wiley.

[14] Khan Academy. *FOIL Method.* https://www.khanacademy.org/math/algebra/introduction-to-polynomial-expressions/multiplying-binomials-2/e/multiplying_expressions_0.5

[15] Lake Tahoe Community College. *Quotient Rule.* https://www.coursehero.com/file/10366503/The-Product-and-Quotient-Rules/

[16] Lazar, E. P. (2006). "Speeding, terrorism, and teaching to the test." *The Drama Review*, pp 1029-1061.

[17] Lord, T., Shelly, C., & Zimmerman, R. (2007). "Putting Inquiry Teaching to the Test: Enhancing Learning in College Botany," *Journal of College Science Teaching*, 36(7), pp 62-65.

[18] Macpherson, S. (2009). "Teaching Biology or Teaching to the Test? How High Stakes Standardized Tests Impact the Biology Classroom." *International Journal of Learning*, 16(7).

[19] Maloney, K. (2006). "Teaching to the Test." *International Journal of Learning*, 13(6), pp. 19-25.

[20] Marzano, R. J. (2001). *Designing a new taxonomy of educational objectives*. Thousand Oaks, CA:

Corwin Press.

[21] Miller, M. D., & Seraphim, A. E. (1993). “Can Test Scores Remain Authentic When Teaching to the Test?" *Educational Assessment,* 1(2), p. 119.

[22] Pickens, S., Ostwald, S. K., Murphy-Pace, K., & Bergstrom, N. (2010). Systematic review of current executive function measures in adults with and without cognitive impairments. *International Journal of Evidence-Based Healthcare*, *8*(3), 110–125. doi:10.1111/j.1744-1609.2010.00170.x PMID:21199379

[23] Popham, W. J. (2004). "Teaching to the Test: An Expression to Eliminate.” *Educational Leadership,* 62(3), pp 82-83.

[24] Popham, W. J. (2003). *Test Better, Teach Better: The Instructional Role of Assessment,* 1st edition. Boston, MA: Houghton Mifflin Harcourt.

[25] Posner, D. (2004). “What's Wrong with Teaching to the Test?” *Phi Delta Kappan*, 85(10), p. 749.

[26] Purdue University. *Quotient Rule.* https://www.math.purdue.edu/files/academic/courses/2015fall/MA15910/MA15910Lesson16Notes(revsp15).pdf

[27] Reitan, R. M. (1958). Validity of the Trail Making test as an indicator of organic brain damage. *Perceptual and Motor Skills*, *8*(3), 271–276. doi:10.2466/pms.1958.8.3.271

[28] Society of Actuaries, *FM February 2019 Syllabus*, https://www.soa.org/Files/Edu/2019/2019-02-exam-fm-syllabus.pdf

[29] Society of Actuaries, *Exam FM Sample Questions*, https://www.soa.org/Files/Edu/2018/2018-10-exam-fm-sample-questions.pdf

[30] Stroop, J Ridley (1995). "Studies of Interference in Serial Verbal Reactions." *Journal of Experimental Psychology*, 18, pp. 643-662.

[31] Toplack, M. E., West, R. F., & Stanovich, K. E. (2013). Practitioner Review: Do performance-based measures and ratings of executive function assess the same construct? *Journal of Child Psychology and Psychiatry, and Allied Disciplines*, *54*(2), 131–143. doi:10.1111/jcpp.12001 PMID:23057693

[32] Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.

[33] Wikipedia, *FOIL Method,* https://en.wikipedia.org/wiki/FOIL_method

[34] Wolfram Mathworld, SOHCAHTOA, http://mathworld.wolfram.com/SOHCAHTOA.htm

[35] Yazdani, Mohammad A. (2008). "The Gagne – van Hieles Connection: A Comparative Analysis of Two Theoretical Learning Frameworks." *Journal of Mathematical Sciences and Mathematics Education, 3*(1).