On Simulation Method of Small Life Insurance Portfolios

By

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Abstract

A new simulation method is developed for actuarial applications with small number of insured lifes. We use the life settlement portfolios as an example. The example shows that as portfolio sizes increases, the variance of financial result (IRR) reduces at much slower pace than previously thought.

Introduction

The basic assumption for traditional actuarial science is that the pool of insured (N) is large. If the annual mortality rate is q, we could reasonably project the death in the year will be ($N \times q$). If the death benefit is $1, the total death benefit will be $(N \times q)$, and the variance of the total death benefit is $N \times q \times (1-q)$. When the Monte Carlo method is used to study financial impact of the variability of the mortality, for simplicity, it is often assumed that the distribution of the death benefit is normal with mean $(N \times q)$, and variance $N \times q \times (1-q)$. See [2] for an excellent application of the traditional actuarial model used in valuation of securities, which is linked to the mortality risks. In this work, we propose a new method of applying the Monte Carlo method to study mortality for smaller N. As the example shows, the old method significantly underestimated the variance of the mortality impact on the financials.

The life settlement business provided a secondary market for life insurance policies. When a policyholder no longer need the death benefit protection, he/she could surrender the policy for the cash value (account value), or if his/her heath deteriorated, he/she could sell the policy to a life settlement company for a settlement price higher than the cash value (account value). Both policyholder and the life settlement company could benefit from the difference of the expected mortality of this policyholder and the mortality table on which the policy is priced upon. The settlement price is based on the health condition of the insured, and many other factors. After the settlement transition, the settlement company has all the mortality risk. The life settlement business has grown rapidly in recent years, and expects to further grow in the future.

The life settlement company usually has a portfolio of 200-500 policies with a face amount of $1M ~$2M. The attained age of the policyholder are usually 70 and up, with the life expectancy usually less than 12 years. The mortality risk is the dominant risk for a life settlement company's portfolio. The author believes that as the portfolio has 200 or more policies, the traditional actuarial method will not fully capture the risk involved. In fact, our new method, which is more theoretically sound, produced a much different result than the traditional method. The difference of result is more significant when the size of the portfolio increases.

The life settlement and the life insurance business are like any other business. The investment is in the front, and the positive cash flow comes later. Let CF(i) be the cash flow at time i, $i = 0,..n$. CF(0) is the initial negative cash flow, the investment. For the life insurance business, it is the initial expense and commission, initial reserve, etc. For the Life Settlement Company, it is the purchase price, and initial premiums. The internal rate of return (IRR) is the most often used measure of profitability of any cash flows. The IRR is defined as the solution of the equation $\sum \text{CF}(i)/(1+\text{IRR})^i = 0$.

People expected 20% IRR for life settlement business. Recent experience shows that 12% is a more realistic expectation. The mortality risk could be measured by the variance of the IRR. The mortality rate and death are considered random variable, and the Monte Carlo method is applied. In this work, we only considered the mortality risk from the randomness itself. We did not consider the mortality risk from the inaccurate underwriting, medical improvements and other systematic factors.

We will use a life settlement portfolio as an example. To simplify our model, let us assume we have the same policy in the portfolio. The Policy is sold to a 75-year-old male issued with standard risk class. The settlement transaction happens in the third year since issue. The universal life policy is based on one actual policy, currently in the market from an actual company. The life settlement company will pay the minimum premium to keep the policy inforce, and hope to collect the death benefit in near future. Generally speaking, the minimum premium is the premium to cover the COI (cost of insurance) charges and expense charges from year to year. The account value is just a bit above 0.

There are few cases of securitization of the cash flows of the life settlement portfolios. See [3] for some recent research ideas related to life settlement securitization. To rate this type of the security, one of the important factors is the size of the portfolio, which impact the volatility of the cash flow and IRR of the portfolio a great deal. Some rating agencies developed models to rate such securities. For a rating agency perspective on life settlement business and its securitization, see [1]. The author believes that the result of this paper should have some immediate business implications.

The traditional actuarial model shows the rapid reduction of the variance of the IRR as the size of the portfolio increases. The more accurate new model shows that the variance reduces at a much slower pace. The implication is that, while the size of the portfolio is an importance factor, it is not as important as we thought before. A security backed by portfolio of size 100, in terms of IRR, is not much riskier than a security backed by a portfolio of size 200.

The models

Let $q(i)$ be the mortality rate of year (i) since issue. It is the probability of death provided alive at the beginning of the year. Let $l(i)$ be the probability of insured surviving to the end of year (i) , which is the time (i), since issue. We have $l(0) = 1$. The probability of death during the year is $q(i) \times l(i-1)$, and

$$\begin{aligned} l(i) &= l(i-1) - q(i) \times l(i-1) \\ &= l(i-1) \times (1-q(i)) \\ &= (1-q(1)) \times (1-q(2)) \times \ldots \times (1-q(i)) \end{aligned} .$$

Let N be the number of policies in the portfolio. We assume the premium is paid at the beginning of the year and the death benefit is paid at the end of the year. The premium rate per unit of Death Benefit is $\mathrm{P}(i)$ for policy year (i). At time 0, the cash flow is

$$\mathrm{CF}(0) = -(\text{Purchase Price} + N \times \mathrm{P}(1)).$$

At the time (i), we have

$$\mathrm{CF}(i) = N \times l(i-1) \times q(i) - N \times l(i) \times \mathrm{P}(i+1).$$

This is the basic deterministic model.

To apply the Monte Carlo method, we simulate the $q(i)$ using the random variable $Q(i)$. The random variable $Q(i)$ has mean $q(i)$, and standard deviation

$$\sqrt{(q(i)(1-q(i))/(\max(1,\ N \times l(i-1)))}.$$

Traditionally, $Q(i)$ is approximated with a normal distribution, and floored at 0.

In our new model, we will not model the decrements like in the traditional actuarial models with q's and l's. We will directly model the timing of the death of each insured in the portfolio. Let $X(j)$ be the year of death of j^{th} policy in the portfolio. The cash flow at the time (i) for this policy (j) will be

$$\mathrm{CF}(i,j) = \begin{cases} 0, & i > X(j); \\ 1, & i = X(j); \\ \mathrm{P}(i+1), & i < X(j). \end{cases}$$

The total cash flow of the portfolio

$$\mathrm{CF}(i) = \sum_{j \ge 1}^{j \le N} \mathrm{CF}(i,j) \ .$$

This new model is more in line with the reality.

We simulate the $X(j)$ as follows. First we generate a sequence of random numbers $\{R(i, j)\}$, which are from uniform distribution [0,1]. Then $X(j)$ is determined as the smallest integer such that $R(X(j), j) < q(X(j), j)$, where $q(i, j)$ is the mortality rate of policy (j) at policy year (i).

The result

We assume the basic policy in the portfolio has a face amount of $2M. Minimum premium is paid to keep the policy stay inforce. The price of the settlement is solved so that in the deterministic actuarial model, the IRR is 14%. We assume the settlement is in policy year 3 since issue. The policies are underwritten at table rating 5 at settlement, i.e. the mortality is 225% of the standard 2001 valuation basic table.

We use the MS excel with macros writing in Visual Basic Application as our software platform. The simulations are run until the variance is stable. When the sample variance is stable, it will be close to the population variance, and the sample statistic will be close to their "true" values for the population. The principle is widely accepted in the actuarial community performing stochastic analysis. With this principle in mind, we run 1500 simulations on the traditional actuarial model, and 5000 simulations on the new method.

The traditional actuarial model produced the following results. The variance is 4.6% when portfolio size is 50 and 1.5% when the portfolio size is 400. This is a significant reduction.

Traditional Actuarial Method				
Size of Portfolio	50	100	200	400
Variance of IRR	4.6%	3.2%	2.2%	1.5%
95% percentile	23.5%	20.2%	18.0%	16.6%
5% percentile	8.9%	9.9%	11.1%	11.8%

The new simulation method produced the following result. We can see that when the size of the portfolio increases, the variance reduced by little. A portfolio of size 100 policies has a similar performance as a portfolio of size 400.

New Simulation Method				
Size of Portfolio	50	100	200	400
Variance of IRR	4.7%	4.2%	4.1%	4.0%
95% percentile	22.0%	20.7%	20.3%	20.2%
5% percentile	6.9%	7.0%	7.0%	7.2%

The reason for the inaccuracy of the traditional model is in the model itself. When the size of the portfolio is small, the fundamental assumption of the traditional actuarial model collapses. New method and thinking needs to be developed. This is clearly the case for life settlement business.

This thinking and approach should have other applications as well. One example could be the decision making process for a life insurance company to accept or reject a large case. A large case is a case with large face amount, and warrants a special consideration. The traditional method, applies mortality rate q to even this one single case, and calculates the IRR for the decision making process. A better and more accurate method should simulate the death of the insured at different policy year, model the cash flow, and determine the risk and return relationship of issuing this particular policy. In recent years, the computing power and the simulation method for financial applications are well developed; it is time to apply those new tools to replace the traditional actuarial logic whenever appropriate.

References

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[3] Charles Stone and Anne Zissu, “Securitization of Senior Life Settlements: Managing Extension Risk” The Journal of Derivatives Spring 2006 (vol. 13, no.3), 66-80.