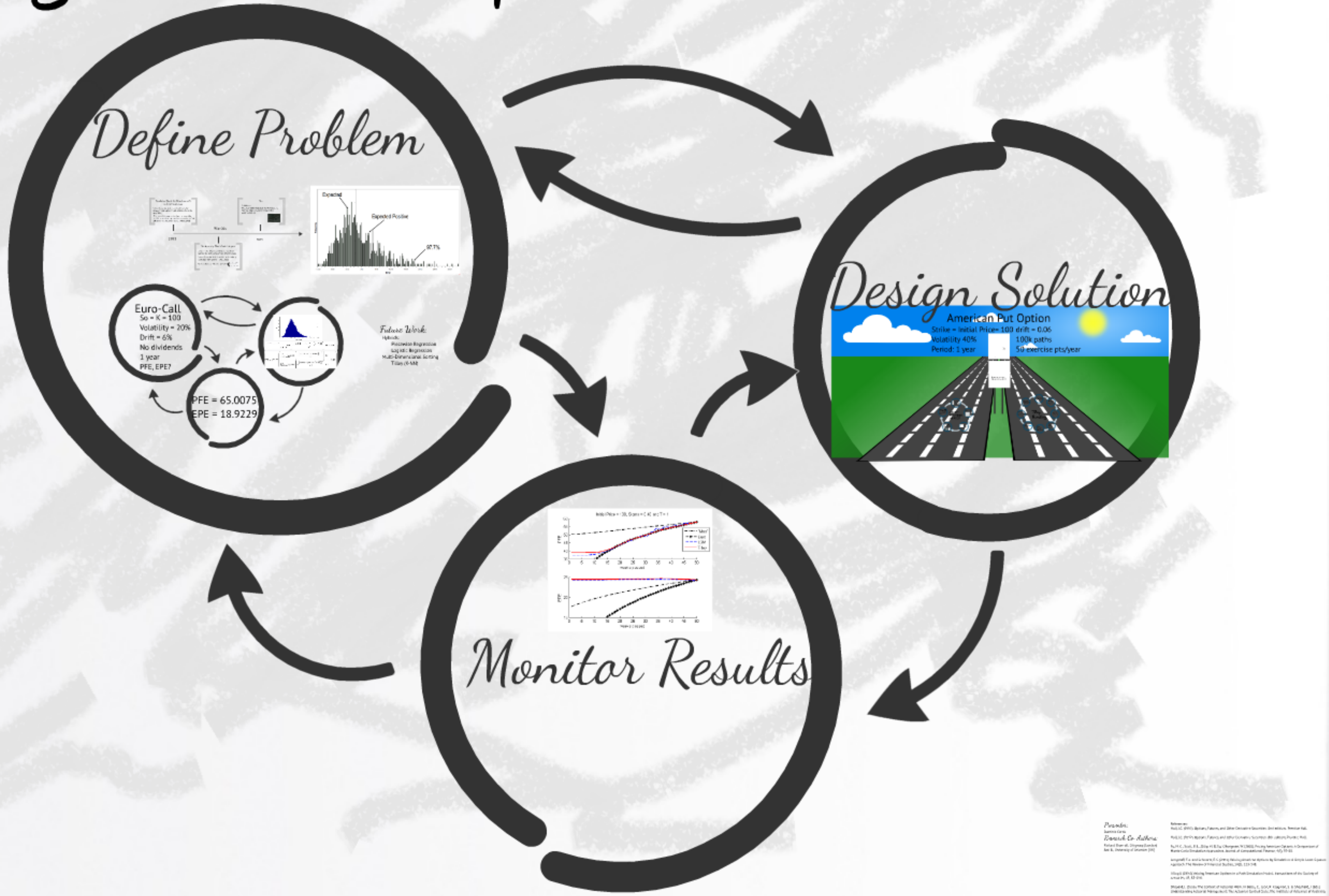
Article from:

ARCH 2014.1 Proceedings

July 31-August 3, 2013


EXTERNAL FACTORS:
Regulatory Environment
Government and judicial context
Physical Environment
Economic and Social Environment
Industry and Business Environment
Sheperd (2006)

Pricing Risk through Simulation: Revisiting Tilley Bundling and Least Squares Monte Carlo Methods

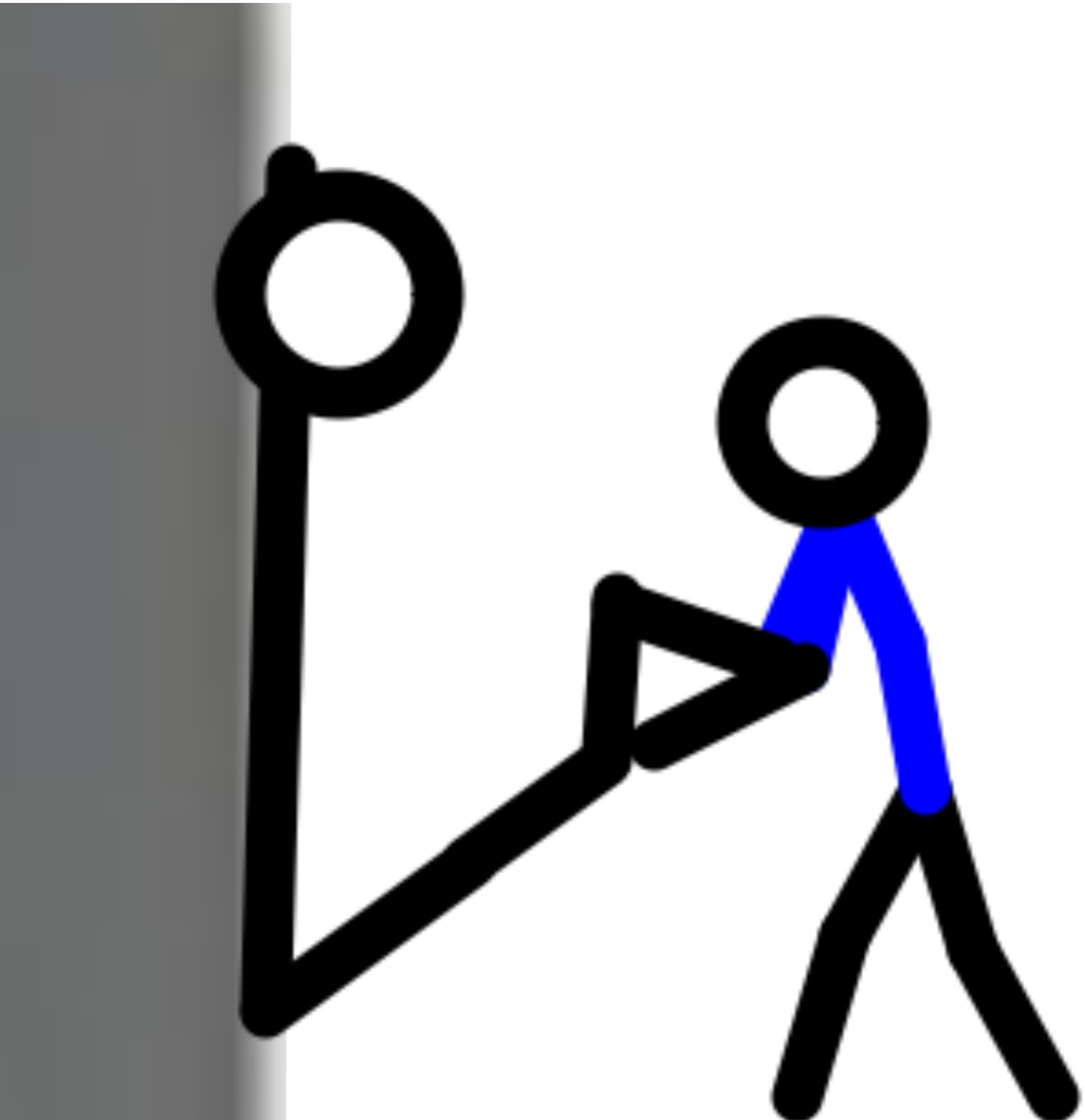


EXTERNAL FACTORS:

Regulatory Environment
Government and judicial context
Physical Environment
Economic and Social Environment
Industry and Business Environment
Sheperd (2006)

Regulatory Environment
Government and judicial context
Physical Environment
Economic and Social Environment
Industry and Business Environment

Sheperd (2006)



Professionalism

Simulation Models for Derivatives with early exercise features

"Monte Carlo simulation can only be used for European-style options." (Hull, 1993 as cited by Fu et al, 2001)

"The goal of this paper is to dispel the prevailing belief that American style options cannot be valued efficiently in a simulation model." (Tilley, 1993)

1993

Development of Monte Carlo techniques

Longstaff and Shwartz develop a simple least-squares approach. (Longstaff and Shwartz, 2001)

"Monte Carlo simulation is well suited to valuing path-dependent options ..." (Hull, 2012)

Fu et al (2001) consider 3 approaches

90s-00s

Now

Risk Metrics
This presentation: Expected Positive Exposure, Potential Future Exposure through Tilley Bundling and LSM

now

Simulation Models for Derivatives with early exercise features

"Monte Carlo simulation can only be used for European-style options." (Hull, 1993 as cited by Fu et al, 2001)

"The goal of this paper is to dispel the prevailing belief that American style options cannot be valued efficiently in a simulation model." (Tilley, 1993)

Development of Monte Carlo techniques

Longstaff and Shwartz develop a simple least-squares approach. (Longstaff and Shwartz, 2001)

"Monte Carlo simulation is well suited to valuing path-dependent options ..." (Hull, 2012)

Fu et al (2001) consider 3 approaches

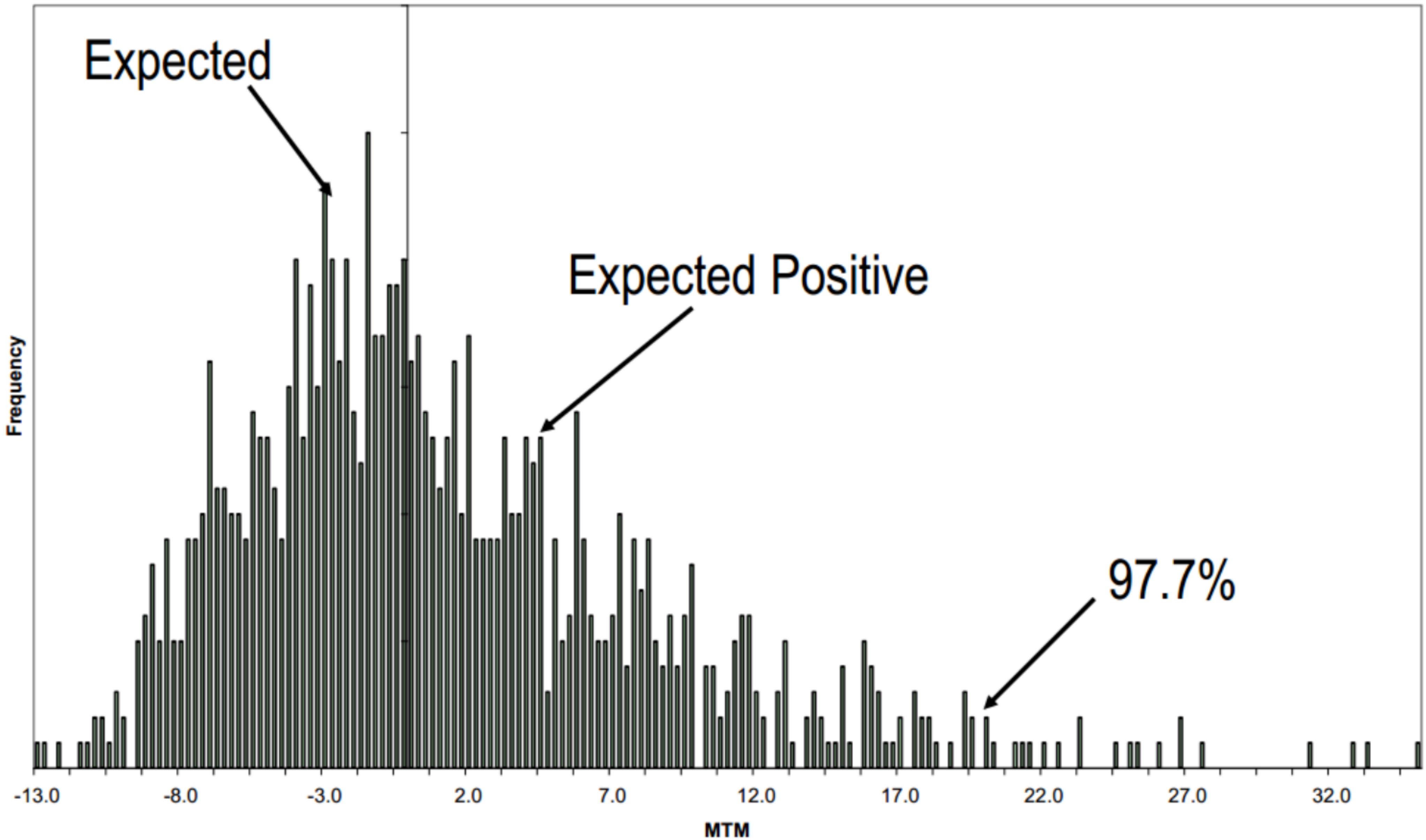
- mimic backwards induction algorithm
- parametrize early exercise curve
- finding efficient upper and lower bounds

- mimic backwards induction algorithm
- parametrize early exercise curve
- finding efficient upper and lower bounds

Now

Risk Metrics
This presentation: Expected Positive Exposure, Potential Future Exposure through Tilley Bundling and LSM

- Basel II/III - Defines EPE, PFE
- Technological progress
- Convergence of financial institutions: risk still there when in the money


Expected
Expected Positive
97.7%
Frequency
-13.0
-8.0
-3.0
2.0
7.0
12.0
17.0
22.0
27.0
32.0
MTM

Euro-Call

So = K = 100

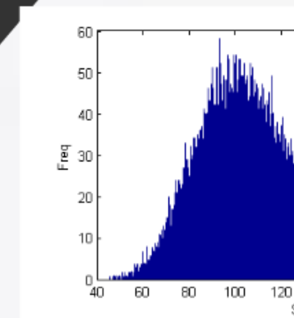
Volatility = 20%

Drift = 6%

No dividends

1 year

PFE, EPE?

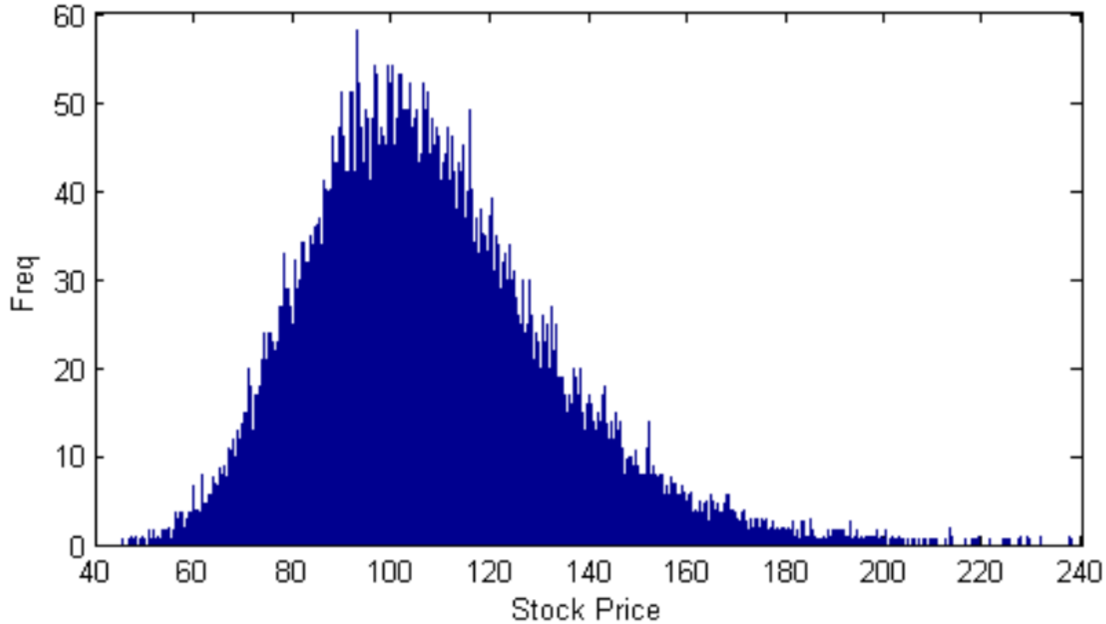


$$\int_0^{e^{0.06}PFE_{0.99}+100} \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{(lo\ldots\right.$$

$$EPE = e^{-0.06}\left(\frac{\int_{100}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}exp\left[\ldots\right.}{\int_{100}^{\infty}\frac{1}{x\sigma\sqrt{2\pi}}exp\left[\ldots\right.}\right.$$

PFE = 65.0075

EPE = 18.9229

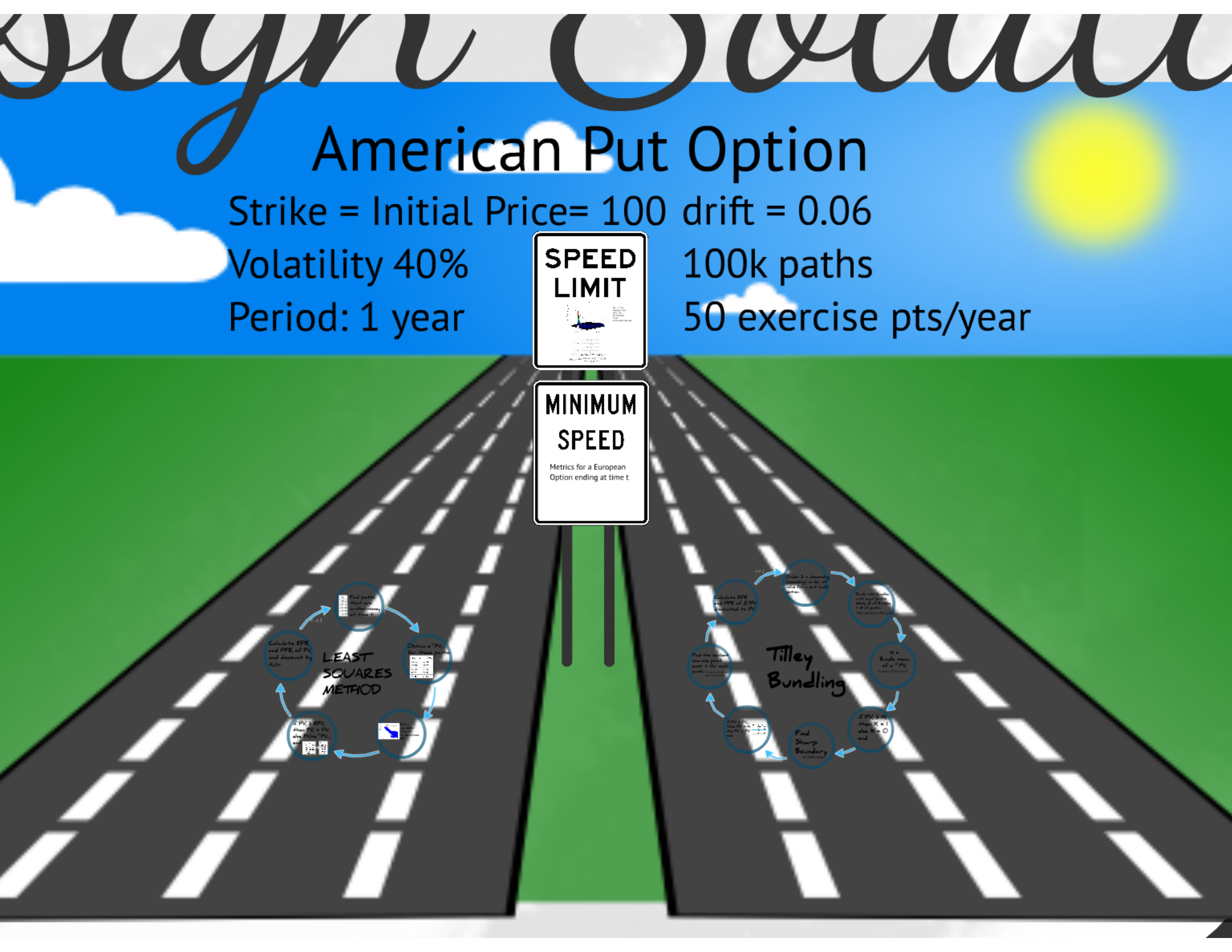


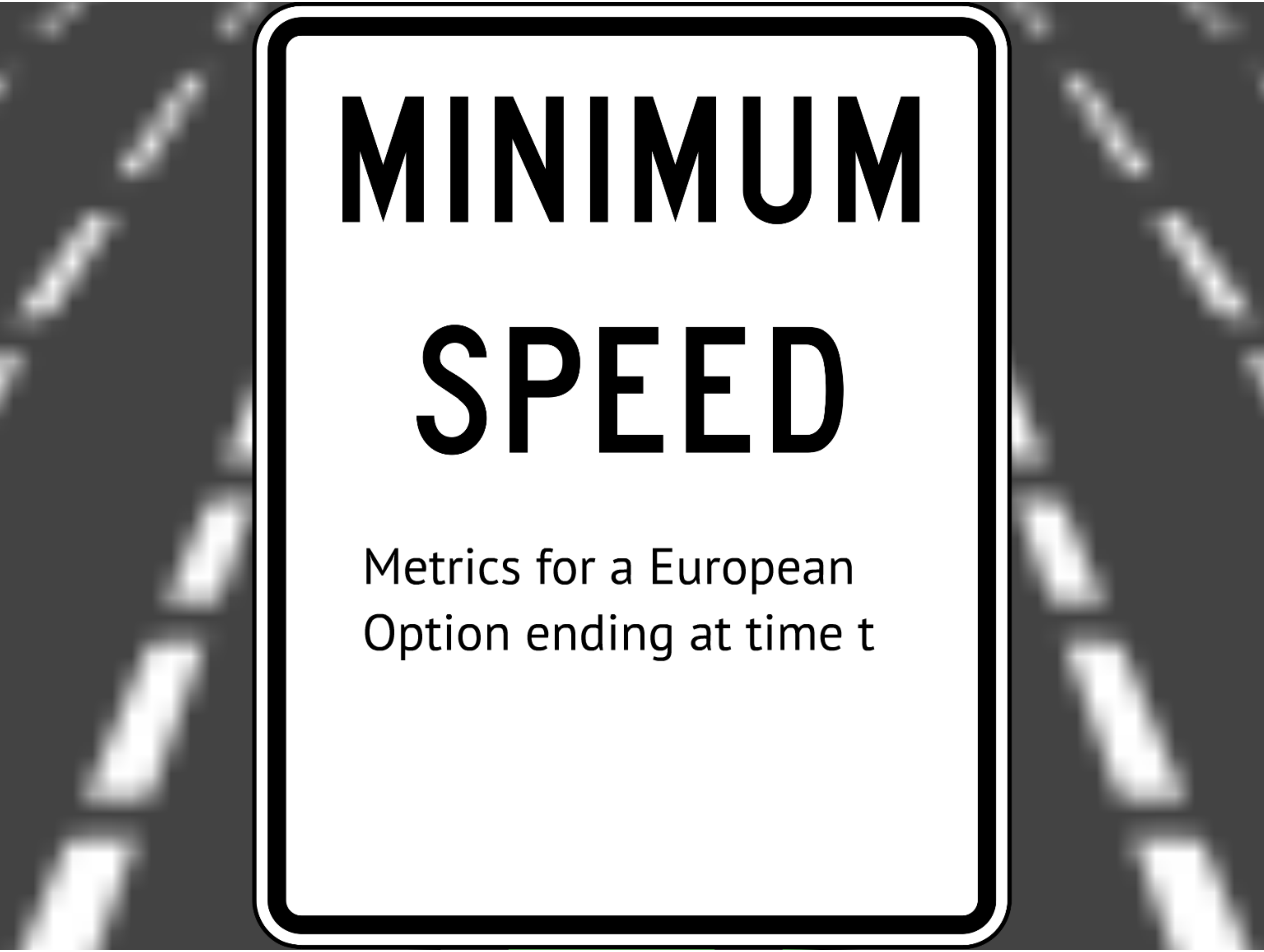
$$\int_0^{e^{0.06}PFE_{0.99}+100} \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{\left(log(x) - log(100) - \left(\mu - \frac{\sigma^2}{2}\right)\right)^2}{2\sigma^2}\right] \mathrm{dx} = 0.99$$

$$EPE = e^{-0.06}\left(\frac{\int_{100}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{\left(log(x)-log(100)-\left(\mu-\frac{\sigma^2}{2}\right)\right)^2}{2\sigma^2}\right] \mathrm{dx}}{\int_{100}^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{\left(log(x)-log(100)-\left(\mu-\frac{\sigma^2}{2}\right)\right)^2}{2\sigma^2}\right] \mathrm{dx}} - 100\right)$$

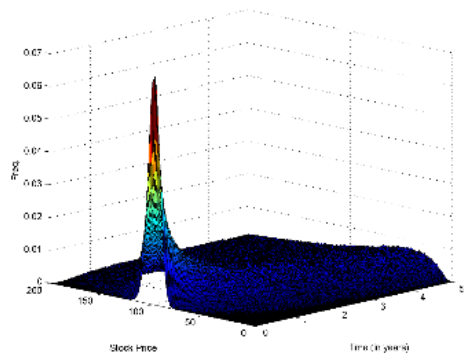
PFE = 65.0075

EPE = 18.9229


American Put Option
Strike = Initial Price= 100
drift = 0.06
Volatility 40%
100k paths
Period: 1 year
50 exercise pts/year
SPEED LIMIT
MINIMUM SPEED
Metrics for a European Option ending at time t
LEAST SQUARES METHOD
Find paths that are in-the-money at time t
Calculate EPE and PFE of PV and discount by
Tilley Bundling
Calculate EPE and PFE of Z*PV discounted to PV
Find the earliest exercise point past it for each path
Find Sharp Boundary
if PV > H then X = 1 else X = 0 end


MINIMUM
SPEED
Metrics for a European
Option ending at time t

SPEED LIMIT



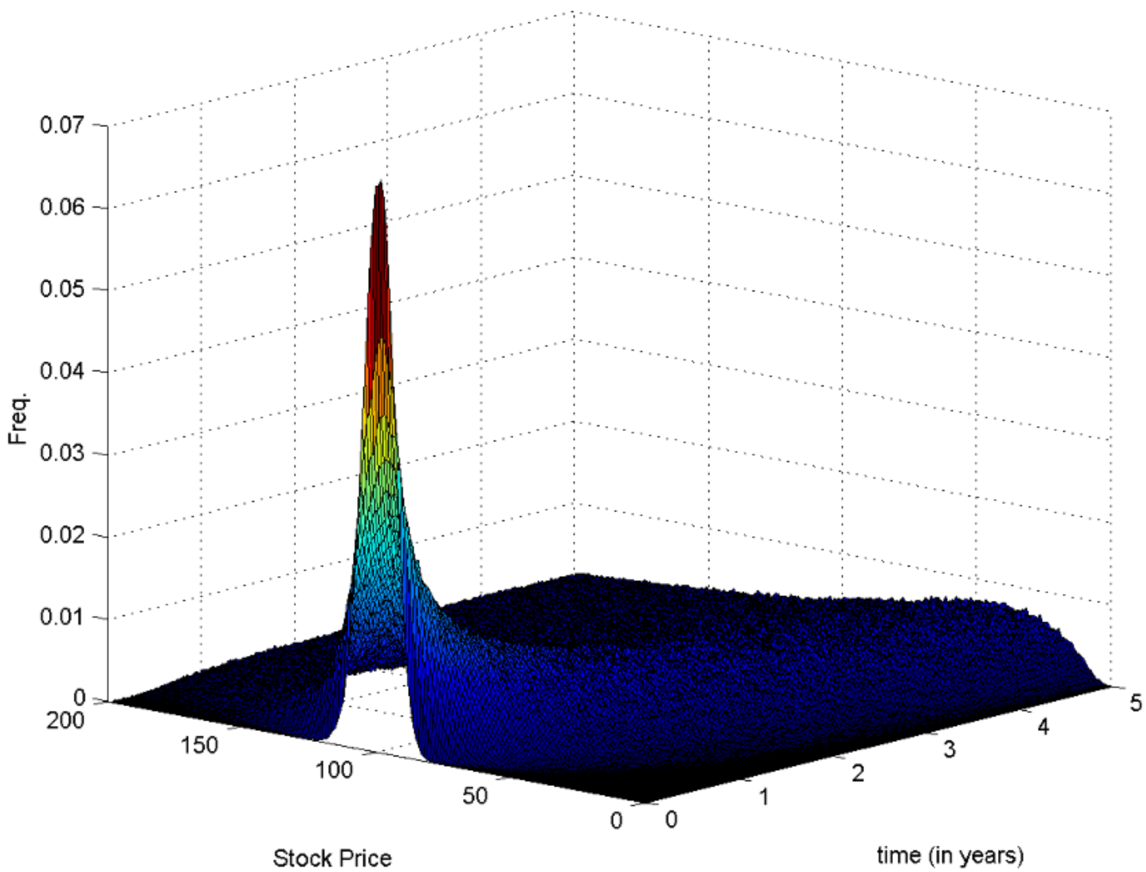
So = K = 100
Volatility = 40%
Drift = 6%
No dividends
5 year
50 exercise times/ year

$$F_2(x, t, a) = \int_0^a \frac{exp\left[-\frac{\left(log(x)-log(S_0)-\left(\mu-\frac{\sigma^2}{2}\right)\frac{t}{50}\right)^2}{\sigma^2\frac{t}{25}}\right]}{x\sigma\sqrt{\pi\frac{t}{25}}}dx$$

$$F_3(x, t, a) = \int_0^a \frac{exp\left[-\frac{\left(log(x)-log(S_0)-\left(\mu-\frac{\sigma^2}{2}\right)\frac{t}{50}\right)^2}{\sigma^2\frac{t}{25}}\right]}{\sigma\sqrt{\pi\frac{t}{25}}}dx$$

$$\sum_{t=k}^{50} F_2(x, t, 100 - PFE_{0.99}e^{0.0012t}) = 0.01(51-k)$$

$$EPE_{k^{th}\ time} = \frac{\sum_{t=k}^{50} e^{-0.0012t}F_2(x,t,100)\left(100-\frac{F_3(x,t,100)}{F_2(x,t,100)}\right)}{(51-k)\sum_{t=k}^{50}F_2(x,t,100)}$$



So = K = 100
Volatility = 40%
Drift = 6%
No dividends
5 year
50 exercise times/ year

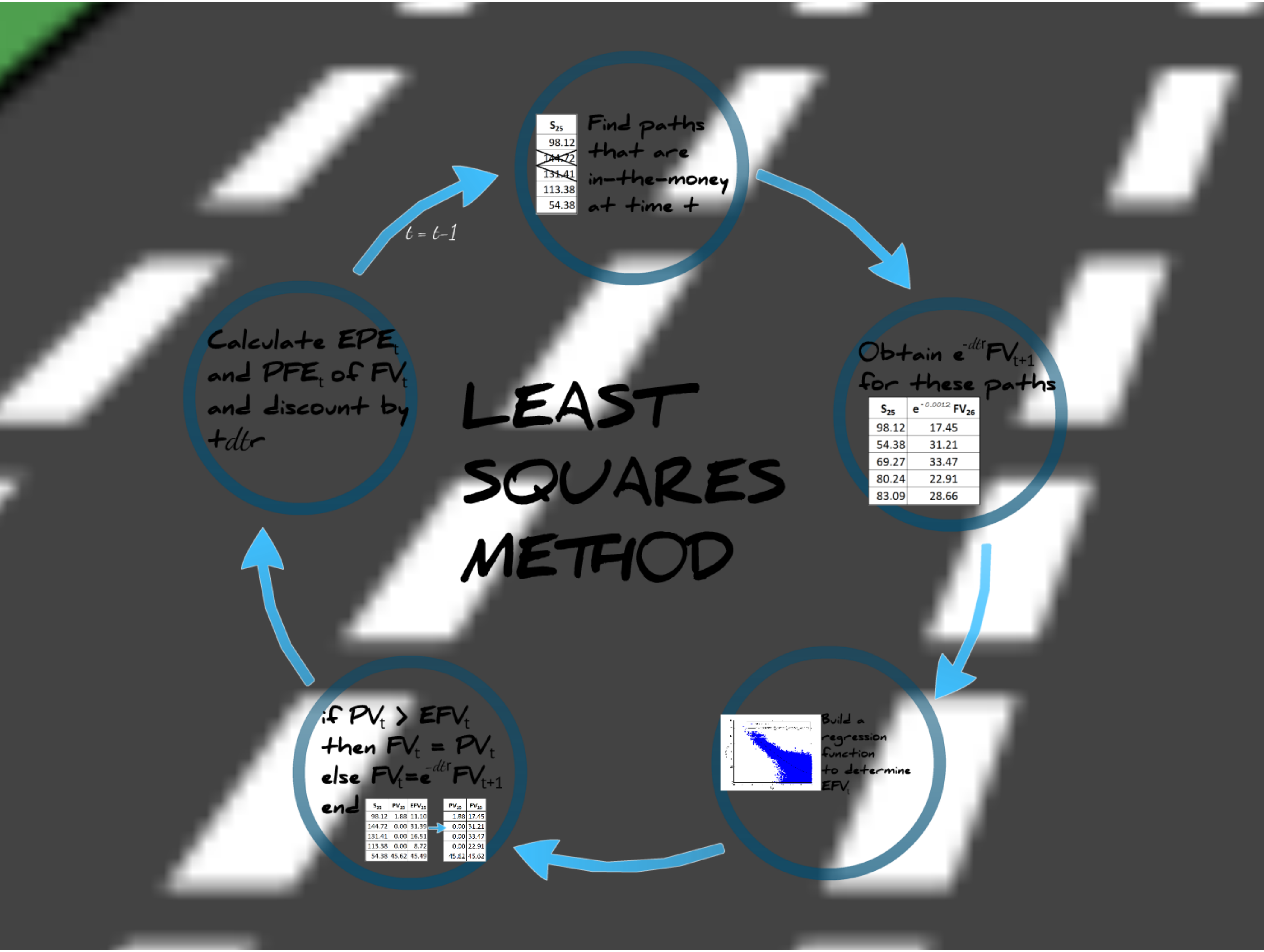
$$F_0(x,t,a) = \int^{a} \frac{exp\left[-\frac{\left(log(x)-log(S_0)-\left(\mu-\frac{\sigma^2}{2}\right)\frac{t}{50}\right)^2}{\sigma^2\frac{t}{25}}\right]}{\ldots} dx$$

$$F_2(x,t,a) = \int_0^a \frac{exp\left[-\frac{\left(log(x)-log(S_0)-\left(\mu-\frac{\sigma^2}{2}\right)\frac{t}{50}\right)^2}{\sigma^2\frac{t}{25}}\right]}{x\sigma\sqrt{\pi\frac{t}{25}}}\,\mathrm{d}x$$

$$F_3(x,t,a) = \int_0^a \frac{exp\left[-\frac{\left(log(x)-log(S_0)-\left(\mu-\frac{\sigma^2}{2}\right)\frac{t}{50}\right)^2}{\sigma^2\frac{t}{25}}\right]}{\sigma\sqrt{\pi\frac{t}{25}}}\,\mathrm{d}x$$

$$\sum_{t=k}^{50} F_2(x,t,100-PFE_{0.99}e^{0.0012t}) = 0.01(51-k)$$

$$EPE_{k^{th}\ time} = \frac{\sum_{t=k}^{50} e^{-0.0012t} F_2(x,t,100)\left(100-\frac{F_3(x,t,100)}{F_2(x,t,100)}\right)}{(51-k)\sum_{t=k}^{50} F_2(x,t,100)}$$



S_{25}
98.12
~~144.72~~
~~131.41~~
113.38
54.38

S_{25}	$e^{-0.0012} FV_{26}$
98.12	17.45
54.38	31.21
69.27	33.47
80.24	22.91
83.09	28.66

S_{25}	PV_{25}	EFV_{25}
98.12	1.88	11.10
144.72	0.00	31.39
131.41	0.00	16.51
113.38	0.00	8.72
54.38	45.62	45.49

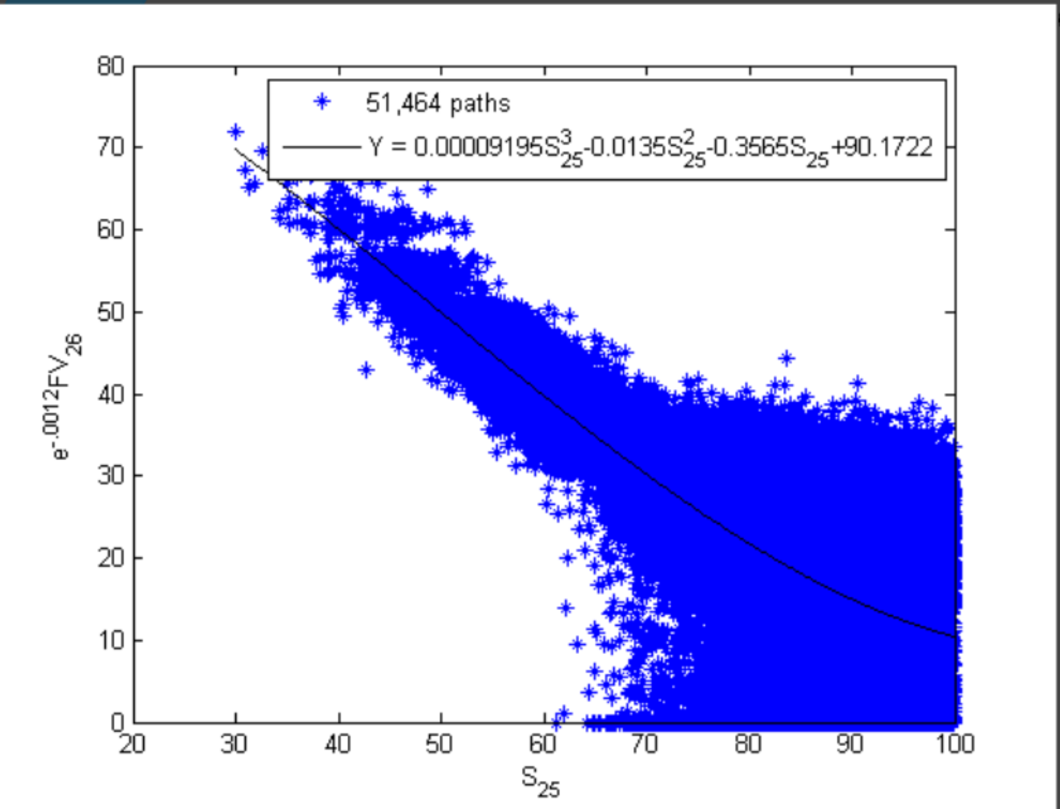
PV_{25}	FV_{25}
1.88	17.45
0.00	31.21
0.00	33.47
0.00	22.91
45.62	45.62

S_{25}
98.12
~~144.72~~
~~131.41~~
~~113.38~~
54.38

Find paths that are in-the-money at time t

Obtain $e^{-dtr}FV_{t+1}$ for these paths

S_{25}	$e^{-0.0012}FV_{26}$
98.12	17.45
54.38	31.21
69.27	33.47
80.24	22.91
83.09	28.66



Build a regression function to determine EFV_t

if $PV_t > EFV_t$

then $FV_t = PV_t$

else $FV_t = e^{-dt\,r} FV_{t+1}$

end

S_{25}	PV_{25}	EFV_{25}
98.12	1.88	11.10
144.72	0.00	31.39
131.41	0.00	16.51
113.38	0.00	8.72
54.38	45.62	45.49

PV_{25}	FV_{25}
1.88	17.45
0.00	31.21
0.00	33.47
0.00	22.91
45.62	45.62

S_{25}	PV_{25}	EFV_{25}
98.12	1.88	11.10
144.72	0.00	31.39
131.41	0.00	16.51
113.38	0.00	8.72
54.38	45.62	45.49

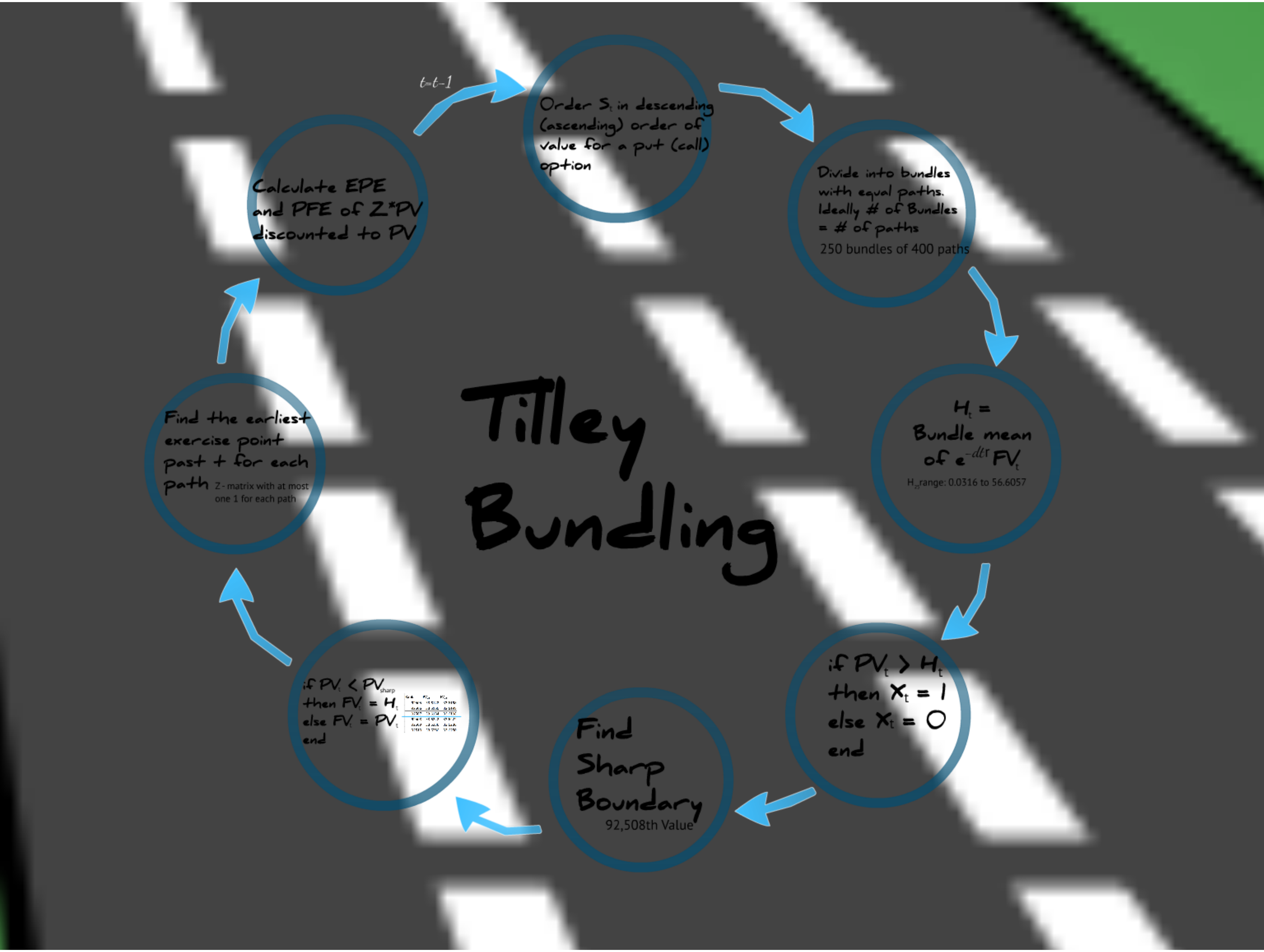
PV_{25}	FV_{25}
1.88	17.45
0.00	31.21
0.00	33.47
0.00	22.91
45.62	45.62

Calculate EPE_t and PFE_t of FV_t and discount by $+dtr$

S_{25}
98.12
~~144.72~~
~~131.41~~
113.38
54.38

$t = t-1$

Calculate EPE_t and PFE_t of FV_t and discount by


Tilley Bundling
t=t-1
Order S_t in descending (ascending) order of value for a put (call) option
Divide into bundles with equal paths. Ideally # of Bundles = # of paths
250 bundles of 400 paths
H_t = Bundle mean of e^{-dtr} FV_t
H_25 range: 0.0316 to 56.6057
if PV_t > H_t then X_t = 1 else X_t = 0 end
Find Sharp Boundary
92,508th Value
if PV_t < PV_sharp then FV_t = H_t else FV_t = PV_t end
Find the earliest exercise point past t for each path
Z-matrix with at most one 1 for each path
Calculate EPE and PFE of Z*PV discounted to PV

Order S_t in descending (ascending) order of value for a put (call) option

Divide into bundles with equal paths. Ideally # of Bundles = # of paths

250 bundles of 400 paths

$H_t =$ Bundle mean of $e^{-dt\,\mathrm{r}} FV_t$

H_{25} range: 0.0316 to 56.6057

if $PV_t > H_t$

then $X_t = 1$

else $X_t = 0$

end

Find Sharp Boundary

92,508th Value

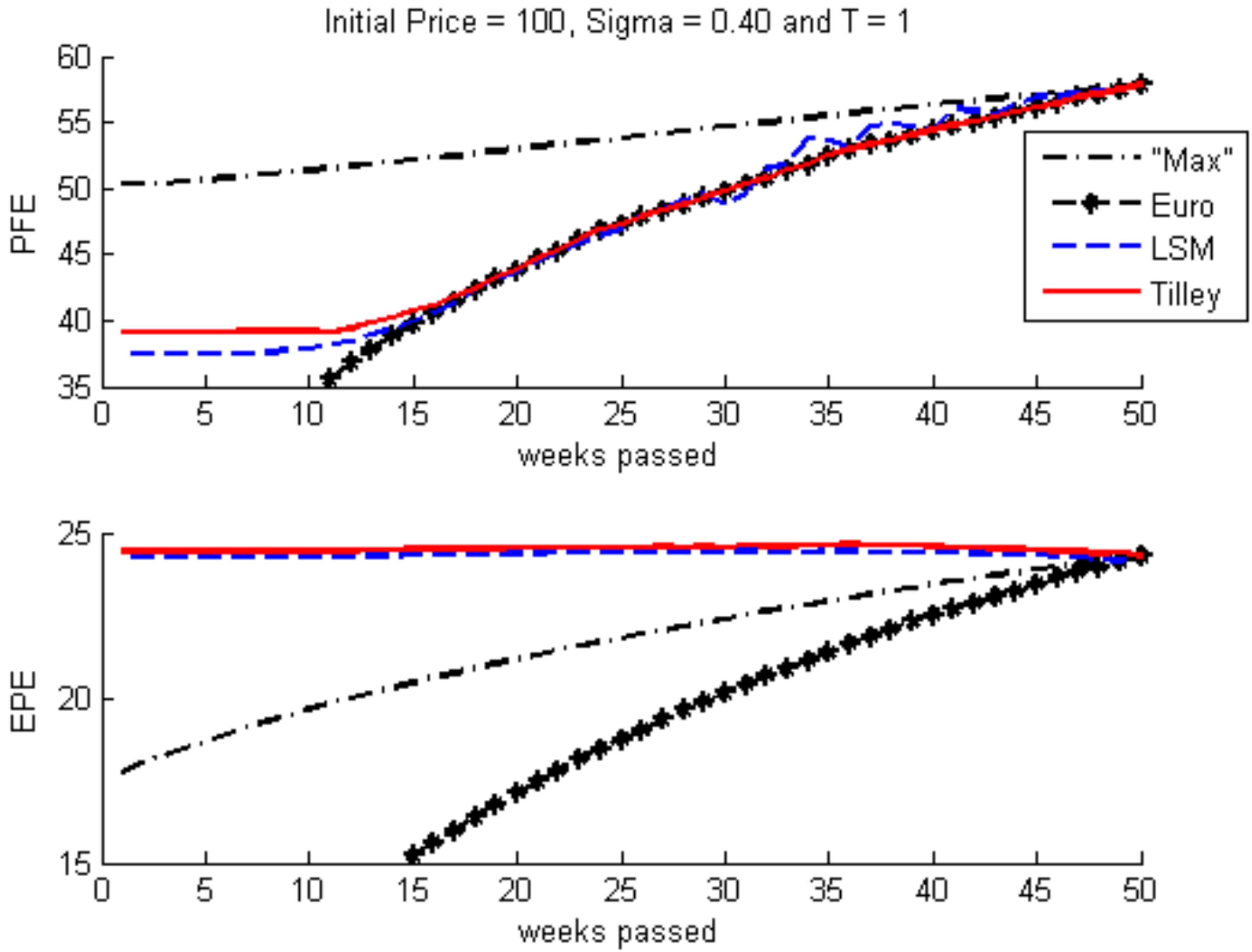
$$\text{if } PV_t < PV_{sharp}$$
$$\text{then } FV_t = H_t$$
$$\text{else } FV_t = PV_t$$
$$\text{end}$$

Rank	PV_{25}	FV_{25}
92505	33.6128	33.6198
92506	33.6160	33.6198
92507	33.6184	33.6198
92508	33.6222	33.6222
92509	33.6236	33.6236
92510	33.6250	33.6250

Find the earliest exercise point past t for each path

Z-matrix with at most one 1 for each path

Calculate EPE and PFE of Z*PV discounted to PV


Initial Price = 100, Sigma = 0.40 and T = 1
PFE
weeks passed
"Max"
Euro
LSM
Tilley
EPE
weeks passed

Future Work:

Hybrids:

- Piecewise Regression
- Logistic Regression

Multi-Dimensional Sorting

- Tilley (K-NN)

Presenter:

Dominic Cortis

Research Co-Authors:

Rickard Branvall, Citigroup (London)
Juxi Li, University of Leicester (UK)

References:

Hull, J.C. (1993). Options, Futures, and Other Derivative Securities (2nd edition). Prentice Hall.

Hull, J.C. (2012). Options, Futures, and Other Derivative Securities (8th edition). Prentice Hall.

Fu, M. C., Scott, B. L., Dilip, M. B, Su, Y, Rongwen, W. (2001). Pricing American Options: A Comparison of Monte Carlo Simulation Approaches. Journal of Computational Finance, 4(3), 39-88.

Longstaff, F.A. and Schwartz, E.S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. The Review of Financial Studies, 14(1), 113-148.

Tilley, J. (1993). Valuing American Options in a Path Simulation Model. Transactions of the Society of Actuaries, 45, 83-104.

Sheperd, J. (2006). The Context of Actuarial Work. In Bellis, C., Lyon, R. Klugman, S. & Shepherd, J. (Eds.) Understanding Actuarial Management: The Actuarial Control Cycle. The Institute of Actuaries of Australia.