Use of Overlapping Generations Model in Modeling Demographic Change

Introduction to the Literature Review

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1 Introduction

One component of this project is to model the impact of population structure on returns across asset classes. Previous research suggests that an overlapping generations (OLG) model is an appropriate modeling approach. This literature review has been prepared to provide background information regarding OLG models and the diversity and complexity that may be introduced into such models.

The accompanying spreadsheet lists relevant papers and characteristics of these papers that may be important to consider in building our model. As well as identifying the authors and the citation for the paper, the spreadsheet provides brief commentary regarding the following characteristics when they appear in a paper:

- Generations
- Utility and Decision
- Labour Supply Choice
- Gender
- Demographic: Fertility, Death and Migration
- Sources of Income
- Asset Classes
- Wage Determination: Ability/Human Capital/ Productivity
- Government Involved in the Pension
- Economy: Open versus Closed
- Conclusion (if relevant)
- Other Remarks

This review has been prepared to provide a fuller discussion of some of the characteristics that may be especially relevant in our decision-making regarding the model we will build.

2 General Background

2.1 Partial Equilibrium Analysis vs. General Equilibrium Analysis

In economic models, there exist clear contrasts between partial and general equilibrium analysis. Partial equilibrium analysis takes into consideration a single market (or a limited number of markets) and assumes that what happens in this market will have no effect on other markets. In contrast, general equilibrium analysis attempts to explain co-movement of variables in the entire economy with several or many interacting markets, all of which impact the other markets through a variety of channels.

Partial equilibrium analysis is usually simpler, easier to implement, and is sufficient when the changes in one market have few repercussions on other markets. However, it is often not reasonable to assume limited interaction between different markets. For example, if we study the impact of a policy that increases the official retirement age on household savings rates, in a partial equilibrium framework, an assumption that wage rates remain constant (that is, not incorporating the effects in the labour market) may lead to a severe over-estimate of the policy impact. As McMorrow and Roeger (1999) point out, the use of a static or partial equilibrium analysis may help identify key relationships, but applying partial equilibrium results to policy decisions can be problematic.

In theory, general equilibrium analysis is preferred over partial equilibrium analysis in the studies of demographic change. However, general equilibrium models are far more complex and often do not have analytical solutions. The equilibrium trajectory of key variables is typically defined by systems of non-linear differential equations representing the inter-dependence of variables in the economy, and must be numerically solved using dynamic optimization algorithms. The practical implementation of these models was only made possible with recent advancements in computing technology, and currently, the literature contains several examples of adaptations of computable general equilibrium models, based on the OLG framework, which study the effect of demographic change in OECD countries (Fougère and Mérette 1999; Aglietta et al. 2007; Beetsma and Bucciol 2009; Muto et al. 2012).

2.2 The Baseline OLG Model Structure

OLG models have been developed to account for complex economic interactions involving more than one generation. Economic models that incorporate demographic transition have the potential

to enhance predictions of actuarial models. Seminal works by Diamond (1965) and Auerbach and Kotlikoff (1987) have popularized OLG models due to their ability to make predictions about important variables such as rates of return on assets, and outcomes of pension restructuring.

The basic mechanism of the OLG model is driven by the life choices of a representative individual, regarding education, labour supply, saving, and retirement, according to a utility function that governs her preferences at any given period and throughout her lifetime. Such a setup permits the model to project the accumulation and transfer of wealth over time and across generations. More complex models involving multiple generations of individuals with heterogeneous life choice preferences can potentially reproduce large movements in asset prices and interest rates (Huffman 1987).

De La Croix and Michel (2002) offer a comprehensive introduction to the setup and derivation of the baseline OLG model, as well as providing guidelines for modeling various extensions. The introduction of De La Croix and Michel (2002) provides insight into OLG models by detailing the underlying assumptions. In this review, we begin with a summary of a basic OLG model, the Diamond (1965) model. Subsequently we provide a broad description of how the literature has expanded the analysis, building on the basic model by incorporating further heterogeneity and consideration of different asset classes. For more detailed derivations, see "A Theory of Economic Growth" by De La Croix and Michel (2002).

The dynamics of the model are illustrated using a simplified version of the Diamond (1965) two-period OLG model, consisting of only firms and households. Individuals supply labour to firms and receive wages in return, then allocate their wages to either consumption or saving. Since individuals are also the owners of firms, savings are invested in terms of capital[1]. In this simple model, the amount of capital created in one period is consumed in full in the next period. Individuals born at time t live for two periods. Let N_t denote the size of the young generation of individuals in period t. Thus at any given time t, the sizes of young and old cohorts living in the economy are given by N_t and N_{t-1}. Moreover, the population is assumed to grow at a constant rate n, *i.e.* $N_t = (1 + n)N_{t-1}$.

During the first stage of their life, individuals are young and they supply one unit of labour to the firm in return for a fixed wage w_t. A young individual allocates her after-tax labour

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[1] Capital is identical to the consumption good produced by firms.

income between consumption c_t and savings s_t, where savings take the form of physical capital that can be used by firms. Thus in period t, a young individual faces the constraint:

$$w_t = c_t + s_t. \tag{1}$$

During period $t + 1$, individuals become old and no longer supply labour when retired. They live off the savings they made when young. In the baseline model, there are no bequests so older individuals consume all their income before they die. In other words, the amount of capital created in period t and its return are fully consumed by the old in period $t + 1$. If r_{t+1} is the rate of return on savings (capital) made in period t, denote $R_{t+1} = 1 + r_{t+1}$ as the total return, the individual's budget constraint in period $t + 1$ is:

$$c_{t+1} = R_{t+1}s_t. \tag{2}$$

Over their lifetime, individuals maximize utility from consumption:

$$\max_{c_t, c_{t+1}, s_t} [u(c_t) + \beta\, u(c_{t+1})], \quad \text{s.t.}\, c_t \geq 0, c_{t+1} \geq 0, \tag{3}$$

together with constraints [1] and [2]. Typically, OLG models incorporate a function where current and future consumption enter additively. The coefficient β represents the "impatience" discount factor. We assume that the utility function $u(\cdot)$ is continuous and twice differentiable in the set of non-negative real numbers, and has characteristics $u'(c) > 0$, $u''(c) < 0$ and $\lim_{c \to 0^+} u'(c) = +\infty$.

By substituting [1] and [2] into the objective function [3], the maximization problem becomes

$$\max_{s_t} [u(w_t - s_t) + \beta\, u(R_{t+1}s_t\,)], \quad \text{s.t.}\, 0 \leq s_t \leq w_t. \tag{4}$$

The optimal saving s_t^* can be derived as a function of labour wage income and return to saving

$$s_t^* = s(w_t, R_{t+1}), \tag{5}$$

by solving the first-order condition

$$u'(w_t - s_t^*) = \beta\, R_{t+1} u'(R_{t+1}s_t^*). \tag{6}$$

Firms use capital and labour to produce consumption goods following the production function $F(K_t, L_t)$. Price of the consumption good is normalized to 1. In this simple model, all markets (goods, labour, and capital) are perfectly competitive. Taking the payments to labour and capital, the wage rate (w_t) and the return factor on capital (R_t) as given, firms solve the profit maximization problem

$$\max_{K_t, L_t}[F(K_t, L_t) - R_t K_t - w_t L_t], \qquad [7]$$

The optimal solutions (K_t^*, L_t^*) are obtained by solving the following first-order conditions

$$F_K(K_t^*, L_t^*) = R_t, \qquad [8]$$

$$F_L(K_t^*, L_t^*) = w_t. \qquad [9]$$

where $F_K(\cdot, \cdot)$ and $F_L(\cdot, \cdot)$ represent the partial derivatives of $F(\cdot, \cdot)$ with respect to capital and labour respectively. In economic terms, the firm's optimal solutions (K_t^*, L_t^*) are obtained by equating the marginal products of inputs with their corresponding costs.

This model will generate the pair of prices (w_t, R_t), and

$$L_t^* = N_t, \qquad [10]$$

$$K_t^* = N_{t-1} s_{t-1}^*. \qquad [11]$$

Then, the dynamics of the OLG model can be characterized by a sequence of the aggregate capital $\{K_t^*\}_{t=1}^{\infty}$ and savings $\{s_t^*\}_{t=0}^{\infty}$, given an initial capital stock $K_0 > 0$.

Extensions to the baseline OLG model include modeling education, gender-specific labour provision, the government sector, public debt, and pension systems. De La Croix and Michel (2002) provide a guide to modeling many of these topics, and it is a strongly recommended read to gain understanding of the various techniques. In the next sections, various extensions are discussed in light of the developments in recent literature.

3 Generation and Cohort Size

Seminal papers (see Allais 1947, Samuelson 1958, and Diamond 1965) explicitly introduce the concept of life-stage or age-specific choice in the OLG model with a two period lifetime (young, old). Extending the two-period model, typically, individuals' lives are divided into more than

two periods, or "stages". Each of the stages is populated by a representative cohort. As an example, a four-period discrete OLG model developed by Buyse et al. (2012) has 3 working and 1 retired cohort, each with distinct education, work and leisure preferences. Further, to allow for more variation in post-retirement behaviour, Magnani and Mercenier (2009) use 8 discrete generations with 5 working and 3 retired cohorts. Extending these models to incorporate multiple retired cohorts has significant implications for analyzing pension structure as it facilitates modeling early retirement and penalties for insufficient contribution (Martín et al. 2010). In addition, it allows for the construction of cohort-specific mortality rates in which individuals are assigned either a fixed or variable probability of death upon entering each retirement stage. Engen et al. (1996) note that the introduction of uncertain lifespan creates alternative savings motives.

The recent literature has extended the analysis to include 70 to 100 generations of age groups. These models calibrate exogenous variables such as fertility rate, age- and cohort-specific mortality rates, and are able to generate predictions of consumption and saving decisions that closely resemble the target economy (e.g. Muto et al. 2012; Beetsma and Bucciol 2009; Cerný et al. 2006). In addition, Kudrna et al. (2014) present a model of age dependencies, in which individuals aged 0 to 20 burden the public finance system with education and healthcare costs, reducing the amount of old-age benefit to be distributed to retirees.

4 Individual Heterogeneity

Introduction of additional individual-level (intra-cohort) heterogeneity makes it possible to generate a greater variety of consumption and savings paths. The most common source of individual-heterogeneity arises by assuming that individuals are born with different levels of innate ability. Ability is then modeled as a productivity factor that affects the individuals' lifetime earning potential. The literature provides several examples, including models of heterogeneous human capital in the OLG framework. In chronological order, examples of some techniques are: a spill-over model in which aggregate human capital is determined by amount of post-secondary education of the economy (Fougère and Mérette 1999); estimating an ability coefficient, as a function of age, gender and education (Heer and Maussner 2006), or by regression on age and gender (Magnani and Mercenier 2009); and assigning arbitrary levels of ability which affect wage at each stage in life (Buyse et al. 2012).

4.1 Child-Bearing and Female Labour Supply

Another common way to incorporate individual-heterogeneity is by modeling the difference in the labour-supply choices between men and women. Absences from the labour market by child-bearing women, and the direct cost of raising children, result in lower life-time consumption and savings of females (Aglietta et al. 2007). Once women are past their fertile age, Brooks (2003) shows that female labour-supply will increase dramatically. Parameters on fertility rates and population growth can be calibrated to resemble the economy of interest, or they can be embedded directly into the model as the outcome of optimal choices by individuals in the economy. For example, Cipriani (2014) incorporates child-bearing directly into the utility function, based on the argument that birth of children brings happiness to parents. Changes to the baseline model can be illustrated by modifying equation [3]. If only young individuals can bear children, the life-time utility would be increased by a parameter γ times the number of children n_t.

$$U(c_t, c_{t+1}, n_t) = u(c_t) + \beta u(c_{t+1}) + \gamma n_t \qquad [12]$$

At the same time, the cost of raising a child would lower consumption in period t by a parameter θ times the number of children.

$$c_t = w_t - s_t - \theta\, n_t \qquad [13]$$

An external shock such as a recession that causes a decline in wage rates will now impact the fertility rate in the economy. The simulation path of consumption and savings would differ from scenarios absent of such shocks. The ability of OLG models to generate these divergent paths of key macroeconomic variables contributes to their usefulness in analyzing demographic changes.

Fertility decisions and financial consequences of child-bearing may be made as a family. A possible extension to gender-heterogeneity models is suggested by Martín et al. (2010). A two-earner household arrives at a child-bearing decision using the outcomes of the household bargaining model (Agarwal 1997). Even when fertility decisions are made unilaterally, female labour supply is significantly lower around peak fertile age.

4.2 Demographic Changed Induced by Immigration

In many countries, net migration is another key determinant of demographic change. For example, in Canada, 20 percent of the population is foreign-born. Many traditional models fail to capture this complexity of population structure. A key issue is that the asset composition, labour-choices, consumption and saving patterns can vary substantially between domestic residents and immigrants. Fougère et al. (2004) suggests a six-region model, each region with its own population composition and labour income. However, in their model, immigrants enter Canada at some age and bring with them the same amount of assets that a domestic resident of the same age would possess.

5 OLG Model with Government and Pension

The OLG model provides a useful framework to analyze the effect of various government policies. In particular, a major concern in many developed countries is the sustainability of the pension system in a society with an aging population.

In the baseline two-period OLG model outlined in section 2.2, various pension schemes may be modeled by introducing a government sector that collects taxes a_t from the young individuals, and re-distributes invested returns, to the old individuals. The expression for capital evolution [11] is modified to include private savings and the reserves of the pension system (De La Croix and Michel 2002)

$$K_t^* = N_{t-1}s_{t-1}^* + N_{t-1}a_{t-1} \qquad [14]$$

Comparing to [5], the optimization problem of the individuals yields

$$s_t^* = s(w_t, R_{t+1}) - a_t \qquad [15]$$

Thus at any given level of capital, savings of the young individual is decreased by the exact amount of the contribution to the pension system, while consumption in each period is unaffected.

Pension systems may be distinguished primarily by the individual to whom the taxes/contributions are re-distributed. It is assumed that in a fully funded system, the invested contributions are returned (at least approximately) to the same individuals when they are old in

period $t+1$. In a pay-as-you-go system, the contributions made by young individuals in period t pay for pension income of the old individuals who live in period t.

The extent to which a pension system is modeled, and the techniques used in the literature, varies substantially depending on the paper's intended purpose. In studies concerning the asset prices and return to assets, it is sufficient to incorporate a simple pay-as-you-go model so that it influences individuals' savings and investment in various assets (Maurer 2011). In some cases, the government adjusts pension contribution based on inflation (Heer and Maussner 2006), or through different tax rates depending on the individual's abilities determined by amount of education (Fougère and Mérette 1999). With single-country models that include multiple regions, the pension can be modeled as some proportion of the weighted average of labour income from all regions (Fougère et al. 2004). More sophisticated models permit retirement decisions to be determined within the model. In such circumstances, the government may choose to penalize early retirement by imposing a penalty for insufficient history of pension contributions over the individual's lifetime (Martín et al. 2010).

5.1 The Role of Government

In reality the pension structure of most OECD countries is far more complex. Aside from government-sponsored welfare programs that guarantee old-age-pension income, many individuals contribute to some form of private retirement savings through their employers. Consequently, models that attempt to assess fully the impact of an aging population benefit from the use of a two-pillar pension system that better captures the dynamics of funds that households receive after retirement. Kudrna et al. (2014) incorporate the main features of the two-pillar pension system used in Australia. Pillar 1 is the government guaranteed pension payment to those with lower income. Pillar 2 is known as the superannuation guarantee, which mandates employers to make superannuation contributions into each employee's superannuation fund. Beetsma and Bucciol (2011) assume a similar pension system, where the contribution to the government-backed social pension reserve is capped and takes no contribution from low-income individuals. Individuals are permitted to contribute, additionally, through private pension funds if their income exceeds a certain threshold.

The role of a private pension fund and the ways it is affected by demographic change is distinguished from that of a government system. Private pension funds resemble firms in a sense

that they maximize expected returns by choosing an investment portfolio of equity and long-term bonds (Beetsma and Bucciol 2011). On the other hand, the government finances public goods such as health care, education and public pension using revenue from its tax-base (Kudrna et al. 2014; Lisenkova et al. 2013; Muto et al. 2012; and others).

Demographic shifts affect both the composition of the government's spending, as well as the sources of its income, such as taxable wage-income. A government operating in deficit must issue debt B_t to finance its spending G_t and repay the interest and principal in subsequent periods. The constraints of the government are

$$B_t + \tau_t = (1 + r_B)B_{t-1} + G_t \qquad [16]$$

where τ_t is tax raised in period t and r_B is the rate of interest payable on government bonds. Individuals' labour income decreases by the amount of tax.

$$w_t - \tau_t = c_t + s_t \qquad [17]$$

As a result of public debt, the amount of private savings and investment in new capital is lower. Notice, however, that this model specification gives rise to another type of asset in which individuals can invest.

$$K_{t+1} = N_t s_t - B_t \qquad [18]$$

6 Return on Assets

A key feature of our modern economy is the ability for individuals to invest directly or indirectly in the financial markets through instruments such as stocks, bonds, houses, infrastructure, and financial derivatives. An increase in the retired population will change the composition of demand for financial products, their valuations, and their rates of return through various channels. An argument for such a channel suggests that when a large portion of the working-age population retires, they sell their assets to provide for retirement, thus driving down the demand for assets and thereby lowering the rates of return (Poterba 2001). However, there is no consensus in the literature regarding the actual direction of movement. Appendix B of Andrews et al. (2014) provides an extensive review of the literature concerning such studies.

Demographic shifts will affect different asset classes with varying levels of impact. Moreover, many studies suggest that the impact of demographic change on the stock market return of any one country will be negligible, due to the internationally integrated nature of the equity market (Cornell 2012). By contrast, demands for housing and bonds are considerably more affected by the composition of and shocks to the local population. The intuition is that, first, as individuals get older they tend to prefer less risky assets to ensure a stable stream of income during retirement (Kulish et al. 2010). Additionally, because homes are not fungible, if a household decides to sell a house to finance retirement there is a single sale at a point in time, unlike financial securities, such as equity and bonds, which can be sold gradually.

Several models in the literature adapt the OLG framework in order to estimate returns to various asset classes. In a calibrated computable OLG model, Beetsma and Bucciol (2009) incorporate various assets directly into an individual's budget constraint. Savings can be invested in a portfolio of short-maturity bonds, equities and housing assets. The composition of the individual's portfolio is calibrated based on the individual's age and cohort. Cerný et al. (2006) use a similar model where individuals' level of debt and choice of investment in equity and safe assets feed into demand for housing. A house is considered both as an investment good and durable consumption good that relieves owners from rent payments. The Cerný et al. (2006) model is, thereby, able to generate demand for houses which can fluctuate depending on returns to various assets, cost of financing a mortgage, and the individual's labour income.

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