GI ADV Model Solutions
Spring 2022

1. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the annual treaty loss ratios including ALAE for each accident year, 2019-2021 (at the 2023 level).

- The trend period is 2 years for accident year (AY) 2021, 3 years for AY 2020 and 4 years for AY 2019.
- The loss development factors are 2.30 for AY 2021, 1.35 for AY 2020 and 1.10 for AY 2019.
- For each AY, the treaty premium is 15% of the subject premium base (i.e., 600,000).

For each AY:

- trended loss is the untrended loss trended by 5% per year for the trend period
- trended loss in the layer is determined from the trended loss
- trended ALAE is the untrended ALAE trended by 5% per year for the trend period
- ALAE covered for the layer is the trended ALAE times the ratio of trended loss in the layer to the trended loss
- developed loss and ALAE for the layer is the trended loss in the layer plus the ALAE covered for the layer times the loss development factor
- the treaty loss ratio is the developed loss and ALAE for the layer divided by the treaty premium

1. Continued

AY	Trended Loss	Loss in Layer	Trended ALAE	Developed Loss + ALAE	Treaty Premium	Treaty Loss Ratio
2019	486,203	236,203	303,877	422,212	600,000	70.4%
2020	1,041,863	500,000	694,575	1,125,000	600,000	187.5%
2021	275,625	25,625	330,750	129,663	600,000	21.6%

(b) Calculate the annual treaty loss ratios including ALAE with the proposed swing plan for each accident year, 2019-2021 (at the 2023 level).

For each AY:

- the loaded amount of loss and ALAE is the developed loss and ALAE for the layer from part (a) loaded by the retro premium factor of 100/80
- the layer swing-rated premium is the loaded amount of loss and ALAE for the layer subject to a minimum of 10% of subject premium (400,000) and a maximum of 20% of subject premium (800,000)
- the revised treaty loss ratio is the developed loss and ALAE for the layer divided by the layer premium

AY	Developed Loss + ALAE	Loaded Loss + ALAE	Swing-rated Premium	Treaty Loss Ratio
2019	422,212	527,765	527,765	80.0%
2020	1,125,000	1,406,250	800,000	140.6%
2021	129,663	162,078	400,000	32.4%

(c) Provide one argument for and one argument against introducing the swing plan.

Commentary on Question:
There are several arguments that could be made for and against the swing plan. Only one of each was required for full credit. The model solution is an example of a full credit solution.

For: It can reduce the volatility in the annual treaty loss ratios.
Against: The treaty loss ratio for the 3-year period combined with swing rating is higher than the treaty loss ratio without swing rating.

2. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Commentary on Question:

This question required the candidate to respond in Excel for parts (c), (d) and (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c), (d) and (e) is for explanatory purposes only.

Solution:

(a) Define "Risk Adjusted" in the context of the Risk Adjusted Discount Technique.

The interest rate used for discounting accounts for the degree of risk in the given cash flow.

(b) State the discount rate (risk-free or risk-adjusted) that should be used with each cash flow.

P: risk-free
E: risk-free
L: risk-adjusted
S: risk-free

(c) The equation for the premium is PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII).

Calculate each of these five values using a trial premium of 150.

Commentary on Question:

PV(cash flow) is the present value of the cash flow, r_F is the risk-free rate and r_A is the risk-adjusted rate.

For P = 150:
PV(P) = P / (1 + r_F)^(1/12) = 150 / 1.042^(1/12) = 149.48661
PV(L) = L / (1 + r_A) = 120 / 1.016 = 118.11024
PV(E) = E = 24
PV(TUW) = [(P – E) t / (1 + r_F)] – [L × t / (1 + r_A)] = 0.70277
PV(TII) = (S + P – E) r_F t / (1 + r_F) = 2.17658

2. Continued

(d) Calculate the premium for this policy. (Note: Using Excel's Goal Seek function is an acceptable approach.)

Commentary on Question:
This question could be answered using one of three approaches: solving for P from the formulas, using Goal Seek so the difference between PV(P) and PV(L) + PV(E) + PV(TUW) + PV(TII) is zero by adjusting P, or using trial and error instead of Goal Seek. The model solution in the Excel solutions spreadsheet used Excel's Goal Seek function. The model solution in this document used the direct solving for P approach.

PV(P) = 0.99658P
PV(L) = 118.11024
PV(E) = 24
PV(TUW) = 0.23992P – 35.28572
PV(TII) = 0.01008P + 0.66507
PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII) so we have
0.74658P = 107.48959
P = 143.97649

(e) Calculate the UPM for this policy.

UPM = 1 – (L + E) / P = –0.00016

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question required the candidate to respond in Excel for parts (c), (d) and (e). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c), (d) and (e) is for explanatory purposes only.

Solution:

(a) State three advantages of using a parametric curve to model development.

- It only requires estimation of a limited number of parameters.
- It does not require evenly spaced dates.
- It provides a smooth development curve.

(b) State one reason the Cape Cod method is generally preferred over the LDF method.

There are fewer parameters to estimate.

(c) Calculate the MLE of *ELR*.

- MLE of *ELR* is the sum of the incremental payments divided by the sum of Onlevel Premium (OLP) times [G(at the end of the interval) minus G(at the beginning of the interval)].
- The end of the interval is “To” months minus 6 if “To” months >=12; otherwise it is “To” months / 2.
- The beginning of the interval is “From” months minus 6 if “From” months >=12; otherwise it is “From” months / 2.

MLE of *ELR*

= [2,500 + 1,800 + … + 5,300] / [10,000 × (0.50568 – 0) + 10,000 × (0.78509 – 0.50568) + … + 18,000 × (0.43694 – 0)] = 24,100 / 38,738.09

= 0.622127

3. Continued

(d) Calculate the value of the loglikelihood function at its maximum.

This is the value of $\ell = \sum_i [c_i \ln(\mu_i) - \mu_i]$.

- c_i is the incremental payment.
- μ_i is the expected increment, which is the MLE of *ELR* times OLP times [G(at the end of the interval) minus G(at the beginning of the interval)].

The value of the loglikelihood function at its maximum is 169,550.97.

(e) Calculate the total reserve for the four accident years combined.

- The total reserve is the total estimated ultimate minus the total payments.
- The total estimated ultimate is the total OLP times the MLE of *ELR*.

Total reserve = 0.622127 × (10,000 + 12,000 + 15,000 + 18,000) – 24,100
= 10,116.97

4. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel for parts (c) to (f). An example of a full credit solution for these parts is in the Excel solutions spreadsheet. The solution in this file for parts (c) to (f) is for explanatory purposes only.

Solution:

(a) State the variance assumption that underlies each set of age-to-age factors.

(i) Variance proportional to square of previous cumulative
(ii) Variance proportional to previous cumulative
(iii) Variance proportional to 1

(b) Two of Venter's six testable implications of assumptions are the linearity of a model and the stability of its age-to-age factors. He suggests looking at plots of residuals to test each of these.

Identify the independent variable for each of these two tests.

Linearity: previous cumulative amount
Stability: time

(c) Determine whether this correlation is significant at the 10% level using a two-sided t-test with a critical value of 2.92.

The sample correlation coefficient, r, is –0.762. It is significant if the absolute value of the T statistic is greater than the t-test critical value, 2.92. The T statistic is $r(2/(1 - r^2))^{0.5} = -1.664$.

The absolute value of the T statistic is not greater than 2.92, so it is not significant.

4. Continued

(d) Demonstrate that the variance for accident year 4 in the center of the matrix has been correctly calculated.

To demonstrate this, one needs to calculate the age-to-age factors, the f_k values and the α_k^2 values for $k >= 4$.

	Age-to-Age Factors			
AY 1	1.193	1.117	1.044	
AY 2	1.106	1.091		
AY 3	1.098			
f_k	1.130	1.104	1.044	
α_k^2	43.233	6.426	0.955	

The f_k values are used to estimate development for AY 2 to AY 4 so we have all of the $c(\text{AY}, k)$ values as follows:

	Development Year			
	4	5	6	7
AY 1	14,891	17,770	19,852	20,727
AY 2	16,779	18,552	20,232	21,124
AY 3	16,451	18,064	19,935	20,814
AY 4	16,974	19,184	21,171	22,104

$$\begin{aligned}\text{Variance for AY 4} = c(4,7)^2 \times [\, & \alpha_4^2 / f_4^2 \times (1/c(4,4) + 1/(c(1,4)+c(2,4)+c(3,4)) + \\ & \alpha_5^2 / f_5^2 \times (1/c(4,5) + 1/(c(1,5)+c(2,5)) + \\ & \alpha_6^2 / f_6^2 \times (1/c(4,6) + 1/(c(1,6))] \\ = 1{,}565{,}012 &\end{aligned}$$

(e) Demonstrate that the covariance between accident years 2 and 3 has been correctly calculated.

Covariance between AY 2 and AY 3 = $c(2,7) \times c(3,7) \times [\alpha_6^2 / f_6^2 / c(1,6)]$
= 19,403.

(f) Calculate the standard error of the overall reserve estimator for all accident years combined.

This equals the square root of the sum of the entries of the covariance matrix of the reserve estimators: $(44{,}235{,}395)^{0.5} = 6{,}651$.

5. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the following for each portfolio and for the two portfolios combined:

(i) Expected losses

(ii) Variance of losses

(iii) Coefficient of variation

- Expected losses = the sum over i of $p(i)$ times the loss to the portfolio for i
- Variance (Var) of losses = the sum over i of $p(i)$ times $(1 - p(i))$ times the square of the loss to the portfolio for i
- Coefficient of variation = the square root of the variance of the losses divided by the expected losses

	Portfolio A	Portfolio B	Portfolio A+B
(i) Expected losses	53,270	65,110	118,380
(ii) Var of losses	107,160,721,750	211,290,600,250	476,840,776,600
(iii) CoV	615%	706%	583%

(b) Calculate the renewal risk load by portfolio using each of the following methods:

(i) Marginal Variance

(ii) Shapley

(iii) Covariance Share

5. Continued

For (i), Risk Load = λ × Change in Var

- Change in Var for A is Var for A+B minus Var for B
- Change in Var for B is Var for A+B minus Var for A

	A	B
Change in Var	265,550,176,350	369,680,054,850
Risk Load	63,732	88,723

For (ii), Risk Load = λ × Shapley Value

- Shapley Value for A is the Var for A plus the Covariance of A and B
- Shapley Value for B is the Var for B plus the Covariance of A and B
- Covariance of A and B is the sum over *i* of loss to A for *i* times loss to B for *i* times $p(i)$ times $(1 - p(i))$

Covariances	A	B
A	107,160,721,750	79,194,727,300
B	79,194,727,300	211,290,600,250
	A	B
Shapley Value	186,355,449,050	290,485,327,550
Risk Load	44,725	69,716

For (iii), Risk Load = λ × (Var + Covariance to Share)

- Covariance to Share for A is the sum over *i* of Covariance(*i*) times loss to A for *i* / loss to A+B for *i*
- Covariance to Share for B is the sum over *i* of Covariance(*i*) times loss to B for *i* / loss to A+B for *i*

	A	B
Var + Cov to Share	172,783,414,690	304,057,361,910
Risk Load	41,468	72,974

(c) Demonstrate for each method in part (b) whether or not the risk load is renewal additive.

The risk load for the portfolio of A and B combined is λ × Var for A and B combined = λ × 476,840,776,600 = 114,442.

5. Continued

For the Marginal Variance method, the total renewal risk load = 63,732 + 88,723 = 152,455. This does not equal the risk load for the portfolio of A and B combined, so the method is not renewal additive.

For the Shapley method, the total renewal risk load = 44,725 + 69,716 = 114,442. This equals the risk load for the portfolio of A and B combined, so the method is renewal additive.

For the Covariance Share method, the total renewal risk load = 41,468 + 72,974 = 114,442. This equals the risk load for the portfolio of A and B combined, so the method is renewal additive.

6. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Define internal systemic risk.

Internal systemic risk refers to the uncertainty arising from the actuarial valuation models used being an imperfect representation of the insurance process as it pertains to insurance liabilities.

(b) Describe how internal systemic risk contributes to correlation effects in an assessment of insurance liability risk margins.

It contributes through the correlation between valuation classes and the correlation between outstanding claim liabilities and premium liabilities.

(c) Describe the three main sources of internal systemic risk:

(i) Specification error

(ii) Parameter selection error

(iii) Data error

(i) The error that can arise from an inability to build a model that is fully representative of the underlying insurance process.

(ii) The error that can arise because the model is unable to adequately measure all predictors of claim cost outcomes or trends in these predictors.

(iii) The error that can arise due to poor data or unavailability of data required to conduct a credible valuation.

6. **Continued**

(d) Identify which main source of internal systemic risk corresponds to each of the following potential risk indicators:

(i) Best predictors have been identified

(ii) Extent, timeliness, consistency and reliability of information

(iii) Knowledge of past processes affecting predictors

(iv) Number and importance of subjective adjustments to factors

(v) Ability to detect trends in key claim cost indicators

(vi) Value of predictors used

(i) Parameter selection error
(ii) Data error
(iii) Data error
(iv) Specification error
(v) Specification error
(vi) Parameter selection error

(e) Provide the reasoning behind using a CoV scale with these two characteristics.

Commentary on Question:
The model solution provides a reasoning for each. Alternative correct responses were acceptable. For example, an alternative correct response for the characteristic "higher for long-tail lines" is that "it is generally more difficult to develop a modelling approach that is representative of the underlying insurance process for long-tail LoBs."

- Nonlinearity: The marginal improvement in outcomes between fair and good modelling infrastructures is less than the marginal improvement between poor and fair modelling infrastructures.
- Higher for long-tail lines: Key predictors are often less stable for long-tail LoBs and past episodes of systemic risk more likely to impair the ability to fit a good model.

7. Learning Objectives:

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(3a) Explain the mathematics of excess of loss coverages in graphical terms.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

Express each of the following quantities using the labels for the nine areas on the graph:

(i) 50
(ii) 100
(iii) Expected loss before trend
(iv) Expected loss after trend
(v) Increased limit factor for 100 before trend
(vi) Increased limit factor for 100 after trend
(vii) Trend factor for basic limit losses
(viii) Trend factor for losses in excess of the basic limit

(i) G + H + I
(ii) D + E + F + G + H + I
(iii) C + F + I
(iv) B + E + H + C + F + I
(v) (F + I) / I
(vi) (E + F + H + I) / (H + I)
(vii) (H + I) / I
(viii) (B + C + E + F) / (C + F)

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4e) Describe considerations involved in pricing property catastrophe covers.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only. "M" represents million in the solution that follows.

Solution:

(a) Calculate the nominal rate on line.

Nominal rate on line = Annual premium / Occurrence limit
= 15M / 100M = 15.0%

(b) Calculate the underwriting loss (excluding expenses) to ABC Reinsurance if a loss fully exhausts the limit.

Annual premium – Occurrence limit + Additional premium
= 15M – 100M + 50% of (100M + 10% of 15M – 15M)
= – 41.75M

(c) Calculate the premium for an equivalent traditional risk cover.

Annual premium – Profit commission
= 15M – (100% – 10%) × 15M × 80%
= 4.2M

(d) Calculate the rate on line for an equivalent traditional risk cover.

Premium / Cover
= 4.2M / (41.75M + 4.2M)
= 9.14%

(e) Calculate the minimum value of *N* that would allow ABC Reinsurance Company to avoid an expected underwriting loss with the finite risk cover.

Check reciprocal of the rate on line for the equivalent traditional risk cover:
1 / 0.0914 = 10.94. Therefore, the minimum value of *N* would be 11.

8. Continued

(f) A further consideration when comparing a traditional risk cover to a finite risk cover is credit risk.

Explain how credit risk affects the comparison.

The reinsurer will need to consider the credit risk of the ceding company because the reinsurer will need to rely upon the ceding company's ability to pay the additional premium in the event of a full loss. At this point, the ceding company may be financially weakened.