LOSS MODELS

WILEY SERIES IN PROBABILITY AND STATISTICS
Established by *Walter A. Shewhart and Samuel S. Wilks*

Editors: *David J. Balding, Noel A. C. Cressie, Garrett M. Fitzmaurice, Geof H. Givens, Harvey Goldstein, Geert Molenberghs, David W. Scott, Adrian F. M. Smith, Ruey S. Tsay*

Editors Emeriti: *J. Stuart Hunter, Iain M. Johnstone, Joseph B. Kadane, Jozef L. Teugels*

The ***Wiley Series in Probability and Statistics*** is well established and authoritative. It covers many topics of current research interest in both pure and applied statistics and probability theory. Written by leading statisticians and institutions, the titles span both state-of-the-art developments in the field and classical methods.

Reflecting the wide range of current research in statistics, the series encompasses applied, methodological and theoretical statistics, ranging from applications and new techniques made possible by advances in computerized practice to rigorous treatment of theoretical approaches. This series provides essential and invaluable reading for all statisticians, whether in academia, industry, government, or research.

A complete list of titles in this series can be found at
http://www.wiley.com/go/wsps

LOSS MODELS

From Data to Decisions

Fifth Edition

Stuart A. Klugman
Society of Actuaries

Harry H. Panjer
University of Waterloo

Gordon E. Willmot
University of Waterloo

WILEY

This edition first published 2019

Edition History
Wiley (1e, 1998; 2e, 2004; 3e, 2008; and 4e, 2012)

Registered Office
John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA

Editorial Office
111 River Street, Hoboken, NJ 07030, USA

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats.

Library of Congress Cataloging-in-Publication Data

Names: Klugman, Stuart A., 1949- author. | Panjer, Harry H., 1946- author. | Willmot, Gordon E., 1957- author.
Title: Loss models : from data to decisions / Stuart A. Klugman, Society of Actuaries, Harry H. Panjer, University of Waterloo, Gordon E. Willmot, University of Waterloo.
Description: 5th edition. | Hoboken, NJ : John Wiley and Sons, Inc., [2018] | Series: Wiley series in probability and statistics | Includes bibliographical references and index. |
Identifiers: LCCN 2018031122 (print) | LCCN 2018033635 (ebook) | ISBN 9781119523734 (Adobe PDF) | ISBN 9781119523758 (ePub) | ISBN 9781119523789 (hardcover)
Subjects: LCSH: Insurance–Statistical methods. | Insurance–Mathematical models.
Classification: LCC HG8781 (ebook) | LCC HG8781 .K583 2018 (print) | DDC 368/.01–dc23
LC record available at https://lccn.loc.gov/2018031122

Cover image: © iStock.com/hepatus
Cover design by Wiley

Set in 10/12 pt TimesLTStd-Roman by Thomson Digital, Noida, India
"Printed in the United States of America"

10 9 8 7 6 5 4 3 2 1

CONTENTS

Part VI Simulation

PREFACE

The preface to the first edition of this text explained our mission as follows:

> This textbook is organized around the principle that much of actuarial science consists of the construction and analysis of mathematical models that describe the process by which funds flow into and out of an insurance system. An analysis of the entire system is beyond the scope of a single text, so we have concentrated our efforts on the loss process, that is, the outflow of cash due to the payment of benefits.

We have not assumed that the reader has any substantial knowledge of insurance systems. Insurance terms are defined when they are first used. In fact, most of the material could be disassociated from the insurance process altogether, and this book could be just another applied statistics text. What we have done is kept the examples focused on insurance, presented the material in the language and context of insurance, and tried to avoid getting into statistical methods that are not relevant with respect to the problems being addressed.

We will not repeat the evolution of the text over the first four editions but will instead focus on the key changes in this edition. They are:

1. Since the first edition, this text has been a major resource for professional actuarial exams. When the curriculum for these exams changes it is incumbent on us to revise the book accordingly. For exams administered after July 1, 2018, the Society of Actuaries will be using a new syllabus with new learning objectives. Exam C (Construction of Actuarial Models) will be replaced by Exam STAM (Short-Term Actuarial Mathematics). As topics move in and out, it is necessary to adjust the presentation so that candidates who only want to study the topics on their exam can

do so without frequent breaks in the exposition. As has been the case, we continue to include topics not on the exam syllabus that we believe are of interest.

2. The material on nonparametric estimation, such as the Kaplan–Meier estimate, is being moved to the new Exam LTAM (Long-Term Actuarial Mathematics). Therefore, this material and the large sample approximations have been consolidated.

3. The previous editions had not assumed knowledge of mathematical statistics. Hence some of that education was woven throughout. The revised Society of Actuaries requirements now include mathematical statistics as a Validation by Educational Experience (VEE) requirement. Material that overlaps with this subject has been isolated, so exam candidates can focus on material that extends the VEE knowledge.

4. The section on score-based approaches to model selection now includes the Akaike Information Criterion in addition to the Schwarz Bayesian Criterion.

5. Examples and exercises have been added and other clarifications provided where needed.

6. The appendix on numerical optimization and solution of systems of equations has been removed. At the time the first edition was written there were limited options for numerical optimization, particularly for situations with relatively flat surfaces, such as the likelihood function. The simplex method was less well known and worth introducing to readers. Today there are many options and it is unlikely practitioners are writing their own optimization routines.

As in the previous editions, we assume that users will often be doing calculations using a spreadsheet program such as Microsoft Excel®.[1] At various places in the text we indicate how Excel® commands may help. This is not an endorsement by the authors but, rather, a recognition of the pervasiveness of this tool.

As in the first four editions, many of the exercises are taken from examinations of the Society of Actuaries. They have been reworded to fit the terminology and notation of this book and the five answer choices from the original questions are not provided. Such exercises are indicated with an asterisk (*). Of course, these questions may not be representative of those asked on examinations given in the future.

Although many of the exercises either are directly from past professional examinations or are similar to such questions, there are many other exercises meant to provide additional insight into the given subject matter. Consequently, it is recommended that readers interested in particular topics consult the exercises in the relevant sections in order to obtain a deeper understanding of the material.

Many people have helped us through the production of the five editions of this text—family, friends, colleagues, students, readers, and the staff at John Wiley & Sons. Their contributions are greatly appreciated.

S. A. Klugman, H. H. Panjer, and G. E. Willmot

Schaumburg, Illinois; Comox, British Columbia; and Waterloo, Ontario

[1]Microsoft® and Excel® are either registered trademarks or trademarks of Microsoft Corporation in the United States and/or other countries.